Metatheorems come in stylized form

For any $e, \xi, \phi, \rho, v, \xi', \text{ and } \rho'$ such that

$$\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle,$$

**FACT**

Last time: “evaluation doesn’t change the set of global variables”
Metatheorems come in stylized form

For any $e, \xi, \phi, \rho, v, \xi', \phi, \rho'$ such that

$$\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle,$$

we have

$$\text{dom } \xi = \text{dom } \xi'.$$

"Evaluation doesn’t change the set of global variables"
Metatheorems are proved by induction

Induction over structure (or height) of derivation trees $\mathcal{D}$

These are “math-class proofs” (not derivations)

Proof

• Has one case for each rule
• Has multiple cases for some syntactic forms
• Assumes the induction hypothesis for any proper sub-derivation (derivation of a premise)
• Template in book (and handout)
Assume the existence of a derivation

Could terminate in any rule!

Base case:

\[ D = \text{LITERAL} \]

\[ \langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle \]

Both sides identical!

\[ \text{dom} \xi = \text{dom} \xi \]
Holds for formal-parameter lookup

Another base case:

\[ D = \frac{x \in \text{dom } \rho}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \downarrow \langle \rho(x), \xi, \phi, \rho \rangle} \]

Both sides identical!

\[ \text{dom } \xi = \text{dom } \xi \]
Inductive case: good sub-derivation

Assignment to formal parameter

\[
\begin{align*}
\mathcal{D} = & \quad x \in \text{dom } \rho \\
& \quad \frac{\mathcal{D}_r}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle}
\end{align*}
\]

By induction hypothesis on \( \mathcal{D}_r \), \( \text{dom } \xi = \text{dom } \xi' \)

Both sides have same domain!
Inductive case: good sub-derivation

True conditional

\[
\begin{align*}
\mathcal{D} = \frac{\{e_1, \xi, \phi, \rho\} \Downarrow \{v_1, \xi', \phi, \rho'\} \quad v_1 \neq 0}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, \rho \rangle \Downarrow \{v_2, \xi'', \phi, \rho''\}}
\end{align*}
\]

By induction hypothesis on \( \mathcal{D}_1 \), \( \text{dom } \xi = \text{dom } \xi' \)

By induction hypothesis on \( \mathcal{D}_2 \), \( \text{dom } \xi' = \text{dom } \xi'' \)

Therefore, both sides have same domain:
\( \text{dom } \xi = \text{dom } \xi'' \)
The only interesting case: assign to global

\[
\begin{align*}
\text{Do both sides have same domain?} \\
\quad \text{• Does } \text{dom } \xi = \text{dom}(\xi' \{ x \mapsto v \}) \text{ ?}
\end{align*}
\]

By induction hypothesis on \( D_r \), \( \text{dom } \xi = \text{dom } \xi' \)

And \( \text{dom}(\xi' \{ x \mapsto v \}) = \text{dom } \xi' \cup \{ x \} = \text{dom } \xi \cup \{ x \} \)

But \( x \in \text{dom } \xi \)! So \( \text{dom } \xi \cup \{ x \} = \text{dom } \xi \)
Next step: Judge this semantics

Bloom’s hierarchy
1. Remember
2. Understand
3. Apply
4. Analyze
5. Evaluate
6. Create

Two questions:
• What’s awkward
• What would help get closer to C/C++?
Your turn: Ideas?

Less awkward?

More C-like?
Location semantics

Simpler naming:

• Name stands for a mutable location
• Location holds values, updated with set
• Function is just another value
Evaluation judgment using store

Judgment $\langle e, \rho, \sigma \rangle \downarrow \langle v, \sigma' \rangle$

- Mappings in $\rho$ never change
- $\rho$ maps a name to a mutable location
- $\sigma$ is the store (contents of every location)

Intuition of the compiler writer:

- $\rho$ models the compiler’s “symbol table”
  “$x$ is in register 4”
- $\sigma$ models the contents of registers and memory

Classic semantic technique
Blank white card: One question

One question about operational semantics
And now, Scheme