Metatheorems come in stylized form

For any $e$, $\xi$, $\phi$, $\rho$, $\upsilon$, $\xi'$, and $\rho'$ such that

$$\langle e, \xi, \phi, \rho \rangle \downarrow \langle \upsilon, \xi', \phi, \rho' \rangle,$$

**FACT**

Exercise: how to say “evaluation doesn’t change the set of global variables”? 
Metatheorems are proved by induction

Induction over structure (or height) of derivation trees $D$

These are “math-class proofs” (not derivations)

Proof
• Has one case for each rule
• Has multiple cases for some syntactic forms
• Assumes the induction hypothesis for any proper sub-derivation (derivation of a premise)
• Template in book (and handout)
Assume the existence of a derivation

Could terminate in any rule!

Base case:

\[
\mathcal{D} = \langle \text{LITERAL}(v), \xi, \phi, \rho \rangle \Downarrow \langle v, \xi, \phi, \rho \rangle
\]

Both sides identical!

\[
\text{dom} \xi = \text{dom} \xi
\]
Holds for formal-parameter lookup

Another base case:

\[ D = \frac{x \in \text{dom} \rho}{\langle \text{VAR}(x), \xi, \phi, \rho \rangle \Downarrow \langle \rho(x), \xi, \phi, \rho \rangle} \]

Both sides identical!

\[ \text{dom} \xi = \text{dom} \xi \]
Inductive case: good sub-derivation

Assignment to formal parameter

\[
\mathcal{D} = \frac{x \in \text{dom } \rho}{\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle} \quad \frac{\mathcal{D}_r}{\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle}
\]

By induction hypothesis on \(\mathcal{D}_r\), \(\text{dom } \xi = \text{dom } \xi'\)

Both sides have same domain!
Inductive case: good sub-derivation

True conditional

\[ \mathcal{D} = \frac{\langle e_1, \xi, \phi, \rho \rangle \downarrow \langle v_1, \xi', \phi, \rho' \rangle \quad v_1 \neq 0 \quad \langle e_2, \xi', \phi, \rho' \rangle \downarrow \langle v_2, \xi'', \phi, \rho'' \rangle}{\langle \text{IF}(e_1, e_2, e_3), \xi, \phi, \rho \rangle \downarrow \langle v_2, \xi'', \phi, \rho'' \rangle} \]

By induction hypothesis on \( \mathcal{D}_1 \), \( \text{dom} \xi = \text{dom} \xi' \)

By induction hypothesis on \( \mathcal{D}_2 \), \( \text{dom} \xi' = \text{dom} \xi'' \)

Therefore, both sides have same domain:

\( \text{dom} \xi = \text{dom} \xi'' \)
The only interesting case: assign to global

\[ \begin{array}{c}
\text{If } x \notin \text{dom } \rho \quad \text{and} \quad x \in \text{dom } \xi \\
\hline
\langle e, \xi, \phi, \rho \rangle \downarrow \langle v, \xi', \phi, \rho' \rangle \\
\hline
\langle \text{SET}(x, e), \xi, \phi, \rho \rangle \downarrow \langle v, \xi' \{x \mapsto v\}, \phi, \rho' \rangle
\end{array} \]

Do both sides have same domain?

- Does \( \text{dom } \xi = \text{dom}(\xi' \{x \mapsto v\}) \) ?

By induction hypothesis on \( \mathcal{D}_r \), \( \text{dom } \xi = \text{dom } \xi' \)

And \( \text{dom}(\xi' \{x \mapsto v\}) = \text{dom } \xi' \cup \{x\} = \text{dom } \xi \cup \{x\} \)

But \( x \in \text{dom } \xi \)!

So \( \text{dom } \xi \cup \{x\} = \text{dom } \xi \)
And now, Scheme