Welcome

COMP 136
Day 21: MCMC

with Metropolis Hastings

CP4: implement!
forward alg HMMs
viterbi alg HMM
~5 hours
~~20 hr~~

Agenda:
1 - quiz3 recap
2 - Q&A

Upcoming Due Dates

- HW4 released! (due Wed at 11:59pm ET)
- CP4 released, due next Tues (that is late deadline)
- Q4 late next week
- HW5, Q5, CP5 soon

details about final

Random Walk sampler
(uses Metropolis MCMC algorithm)

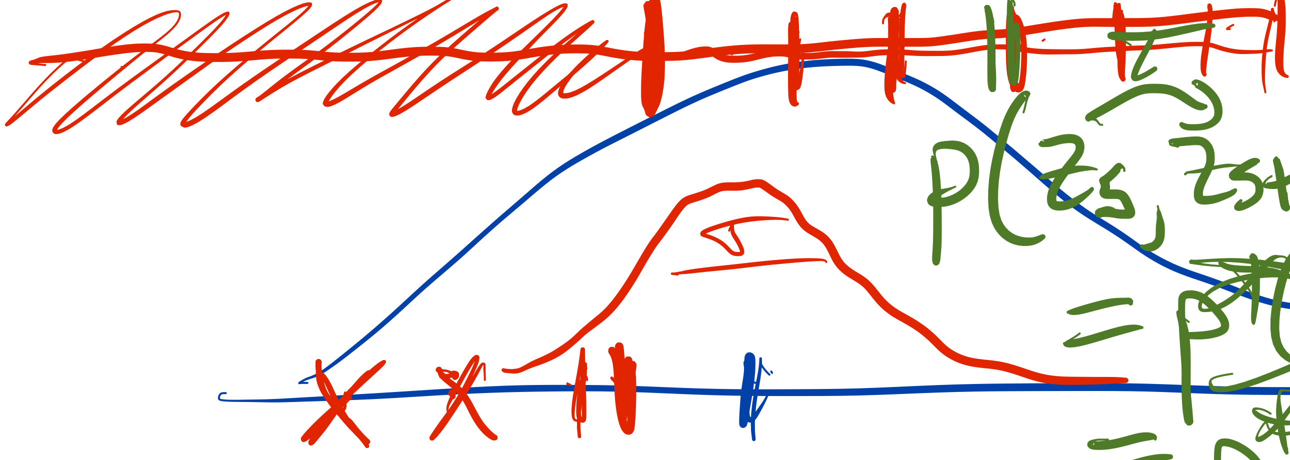
r.v. $z \in \mathbb{R}$ real valued

z_1 initial

$$z' \sim Q(z' \mid z_t) = \mathcal{N}(\overset{\text{mean}}{z_t}, \overset{\overset{0.1}{\downarrow}\ \text{var}}{\sigma^2})$$

$$z_{t+1} = \begin{cases} z' & \text{if } u \leq A(z', z_t) = \frac{p^*(z')}{p^*(z_t)} \\ z_t & \text{otherwise} \end{cases}$$

return $z_1, z_2 \cdots z_S$ s.t. $\boxed{z_S \sim p^*}$



10,000 samples

10^5 samples

$p(z_s, z_{s+1})$?

$\cancel{= p^*(z_s)\, p^*(z_{s+1})}$

$= p^*(z_s)\, \mathcal{T}(z_{s+1} \mid z_s)$

p^*

$z_1, \boxed{z_2, z_3, z_4} \dots, z_S, z_{S+1}, \dots z_{2S} \dots$

z_t not indp. of z_{t-1}

"thinning"
keep every 10^{th} / 100^{th}

with enough samples

$\exists\, S \gg 1$, s.t. $p(z_S) \approx p^*(z)$

if S is large enough

"~~d~~iscard burn-in"
keep after B^{th}

Recall:

cannot eval $p^*(z)$

$\boxed{\text{can eval } \tilde{p}(z)}$

$$\underbrace{p^*(z)}_{\text{valid PDF}} = \boxed{\underbrace{c}_{\text{scalar pos}}\,\underbrace{\tilde{p}(z)}_{\text{scalar pos}}}$$

$$A = \frac{p^*(z')}{p^*(z_t)} = \boxed{\frac{\cancel{c}\,\boxed{\tilde{p}(z')}}{\cancel{c}\,\tilde{p}(z_t)}}$$

accept $\tilde{p}(z') \gg \tilde{p}(z_t)$

accept $0 \leq \sim 1$ $\tilde{p}(z') < \tilde{p}(z_t)$

most likely reject $\tilde{p}(z') \ll \tilde{p}(z_t)$

Usually: what we want target to be is posterior of some r.v. given data

if $\tilde{p}$ is ergodic
$\exists$ one unique p^* stationary

Z: param of model

evaluatable!

Prior: $p(z)$

evaluatable!

Lik: $p(x|z) = \prod_{n=1}^{N} \mathcal{N}(x_n|z, \beta^{-1})$

Poster: $\boxed{p^*(z|x)} = \boxed{\frac{1}{p(x)}}\ \boxed{p(z)\,p(x|z)}$ $\tilde{p}(z)$ $\to$ number

Prior: $z \sim GMM(\boxed{\pi}, \boxed{\mu}, \boxed{\tau})$

Lik: $x_n | z \sim \mathcal{N}(\overset{\text{mean}}{z}, \overset{\text{prec}}{\boxed{\beta^{-1}}})$

Given $x_1 \cdots x_N$, sample $p(z | x_1 \cdots x_N)$

Random Walk accept proba:

$$A: \frac{\tilde{p}(z')}{\tilde{p}(z_t)} = \frac{GMM(z' | \pi, \mu, \tau) \prod_n \mathcal{N}(x_n | z', \beta^{-1})}{GMM(z_t | \pi, \mu, \tau) \prod_n \mathcal{N}(x_n | z_t, \beta^{-1})}$$

2 things you can know:

- eval pdf function ($\tilde{P}$) up to c
- sample from a distribution

NormPDF $\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$ | Exponential PDF ✓

sample?

~~np.random~~

$u \sim \text{Unif}([0,1])$

CDF not analytical invertible

more general

Metropolis

RW

$N(z_t | -)$

Q valid PDF

easy sample

symmetry $Q(z_a | z_b) = Q(z_b | z_a)$

for all z_a, z_b

$$A = \frac{\tilde{p}(z')}{\tilde{p}(z_t)}$$

Metropolis Hastings

Q valid PDF

easy sample

easy eval PDF

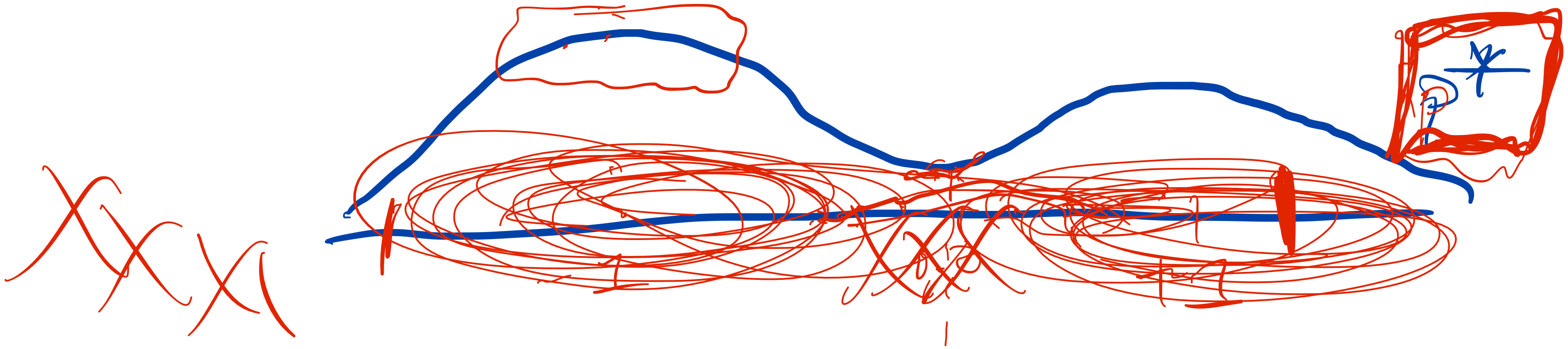
NO SYM REQ'D

$\tilde{p}(a) Q(b|a) = \tilde{p}(b) Q(a|b)$

special case

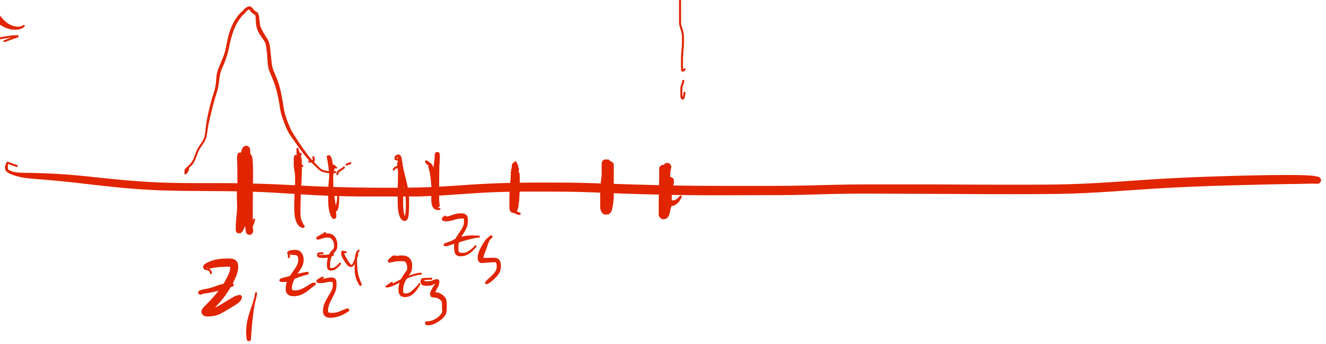
Q from Hamiltonian MC

$$A = \frac{\tilde{p}(z')\, Q(z_t | z')}{\tilde{p}(z_t)\, Q(z' | z_t)}$$



Question: infinite walk thru of RW?

RW with τ small



RW τ large