COMP 150-AVS
Fall 2018

Static Single Assignment Form

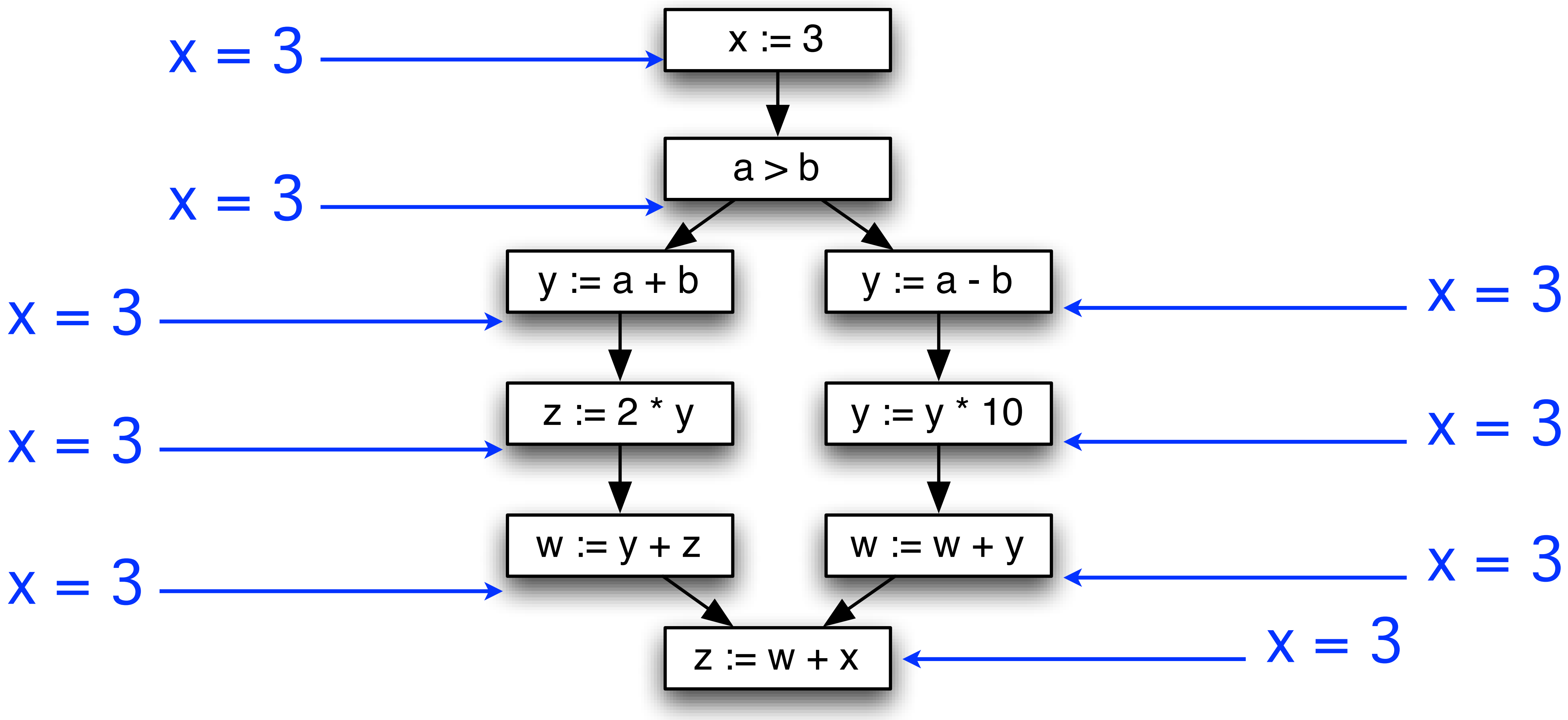
Motivation

- Data flow analysis needs to represent facts at every program point

- What if
  - There are a lot of facts and
  - There are a lot of program points?
  - $\Rightarrow$ potentially takes a lot of space/time

- Most likely, we're keeping track of irrelevant facts

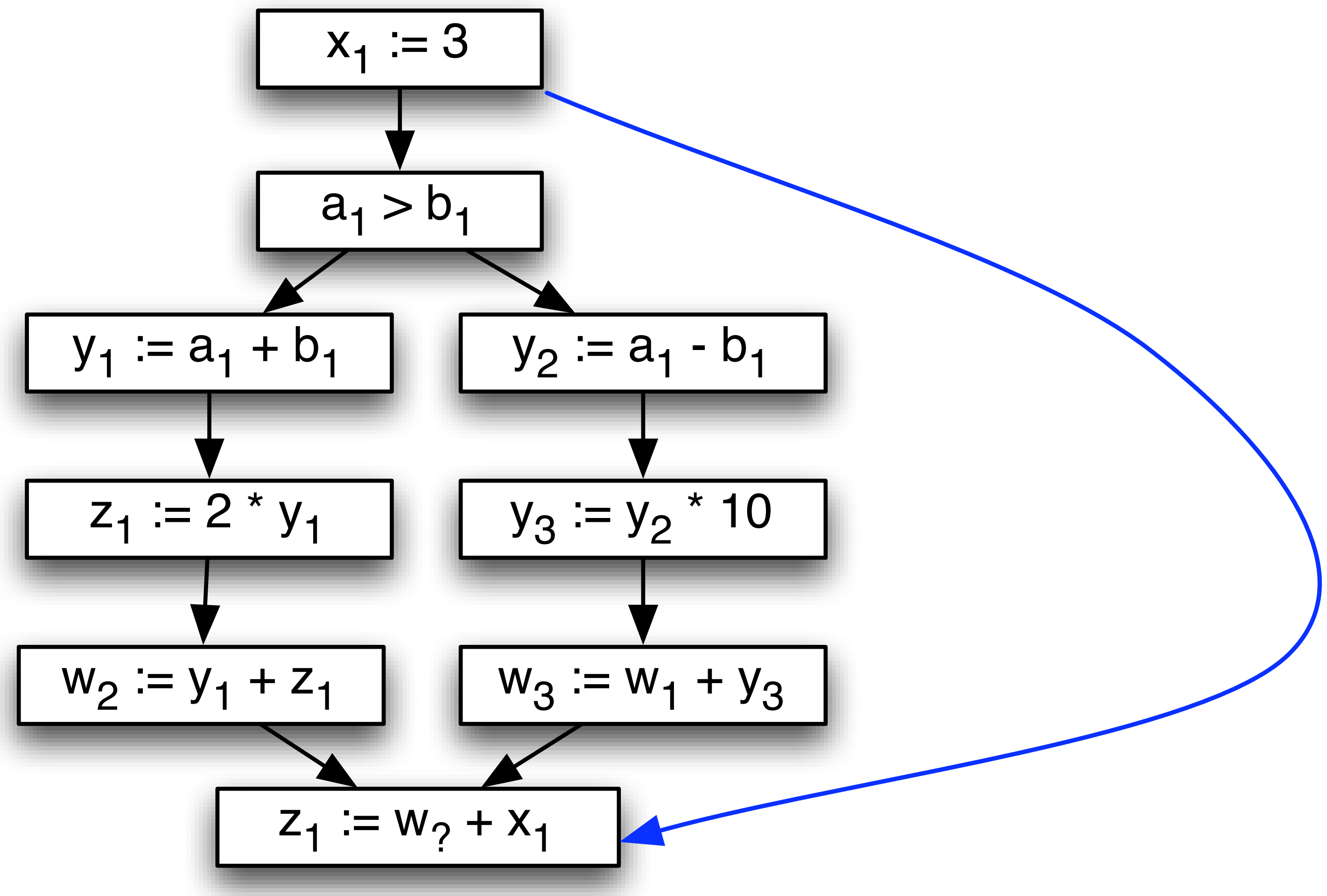
Example



Sparse Representation

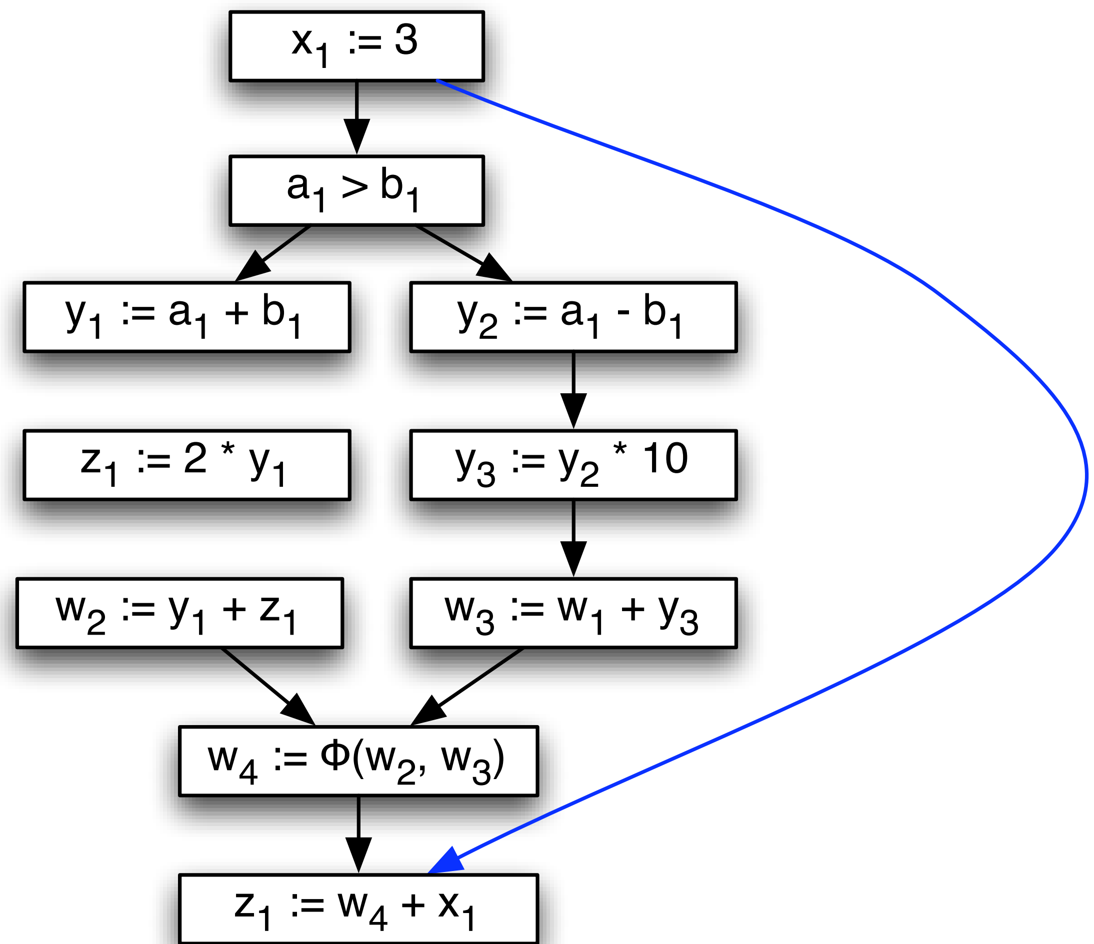
- Instead, we'd like to use a sparse representation
  - Only propagate facts about x where they're needed
- Enter *static single assignment* form
  - Each variable is defined (assigned to) exactly once
  - But may be used multiple times

Example: SSA



- Add *SSA edges* from definitions to uses
  - No intervening statements use/define variable
  - Safe to propagate only along SSA edges

What About Joins?



- Add Φ functions/nodes to model joins
  - Intuitively, takes meet of arguments
  - At code generation time, need to eliminate Φ nodes

Def-Use Chains vs. SSA

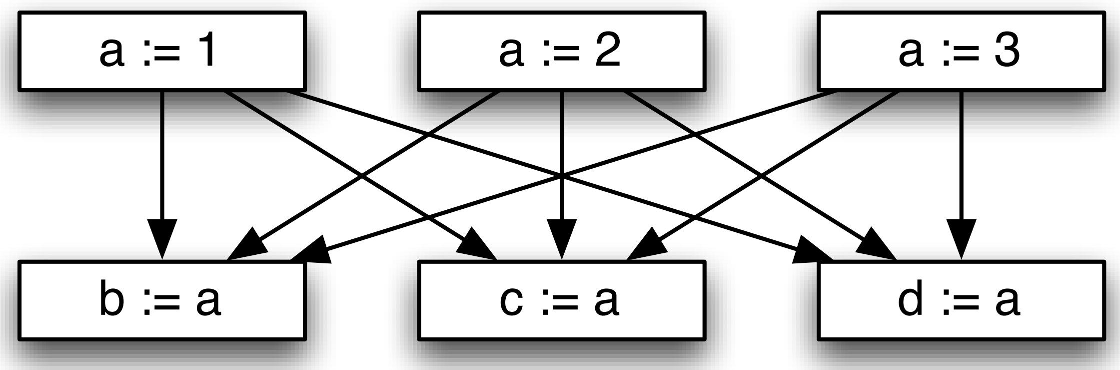
- Alternative: Don't do renaming; instead, compute simple def-use chains (reaching definitions)
  - Propagate facts along def-use chains

- Drawback: Potentially quadratic size

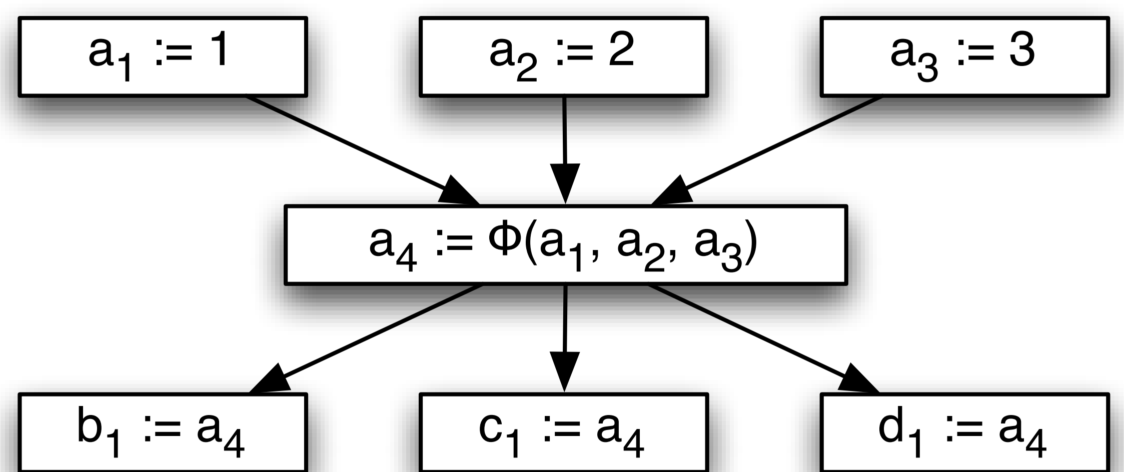
Def-Use Chains vs. SSA (cont'd)

```
case (...) of
  0:  a := 1;
  1:  a := 2;
  2:  a := 3;
end
case (...) of
  0:  b := a;
  1:  c := a;
  2:  d := a;
end
```

Def-Use Chains



SSA Form



Quadratic vs. (in practice) linear behavior

Computing SSA Form

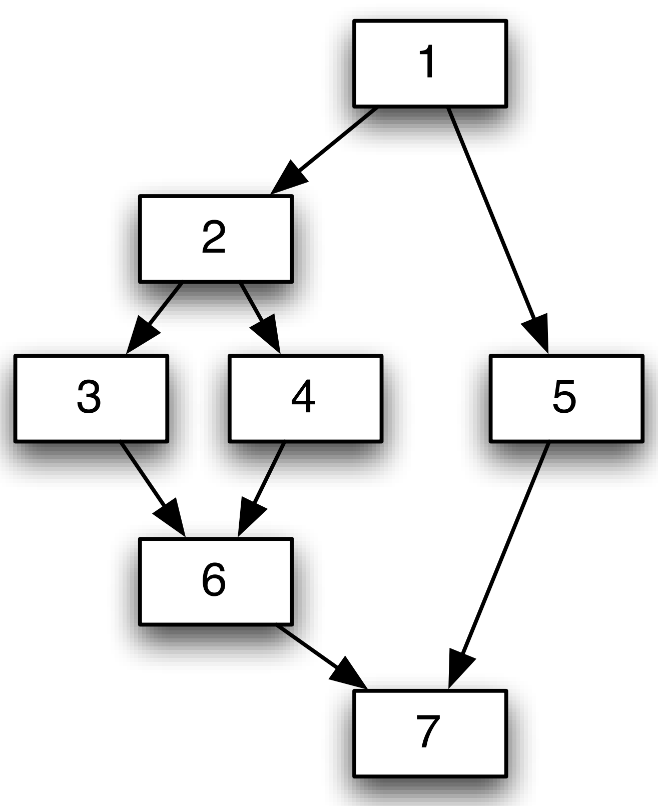
- Step 1: Compute the dominance frontier
- Step 2: Use dominance frontier to place Φ nodes
  - Naive, impractical step 2: put a Φ function for every variable at the beginning of every block
  - Better: If node X contains assignment to a, put Φ function for a in dominance frontier of X
    - Adding Φ fn may require introducing additional Φ fn
- Step 3: Rename variables so only one definition per name

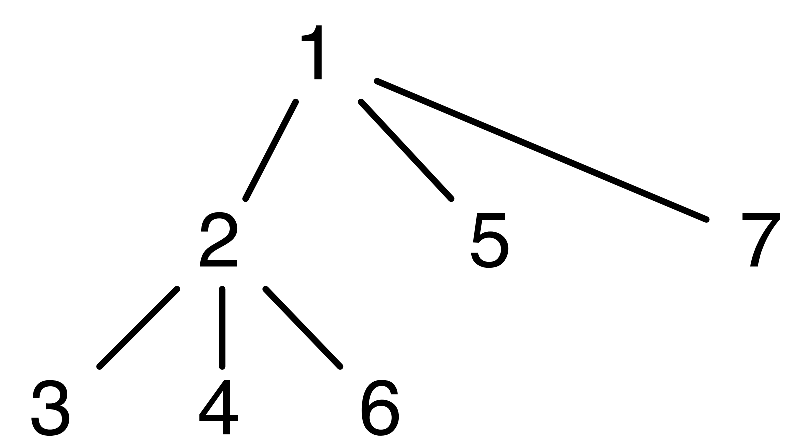
Dominators

- Let X and Y be nodes in the CFG
  - Assume single entry point Entry
- X *dominates* Y (written $X \geq Y$) if
  - X appears on every path from Entry to Y
- Write $X > Y$ when X dominates Y but $X \neq Y$
  - Note $\geq$ is reflexive

Dominator Tree

- The dominator relationship forms a tree
  - Edge from parent to child = parent dominates child
  - Note: edges are not same as CFG edges!





Computing Dominator Tree

- Standard algorithm due to Lengauer and Tarjan

- Runs in time $O(E\alpha(E, N))$
  - E = # of edges, N = # of nodes
  - where $\alpha(\cdot)$ is the inverse Ackerman's function
  - Very slow growing; effectively constant in practice

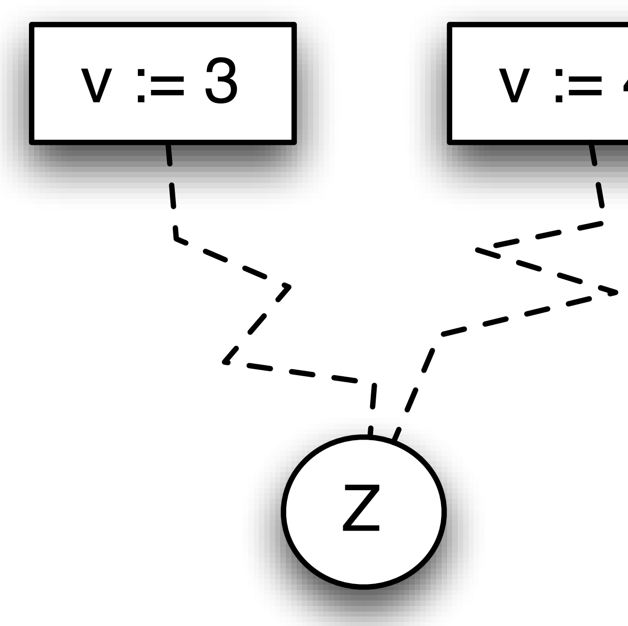
- Algorithm quite difficult to understand
  - But lots of pseudo-code available

Why Are Dominators Useful?

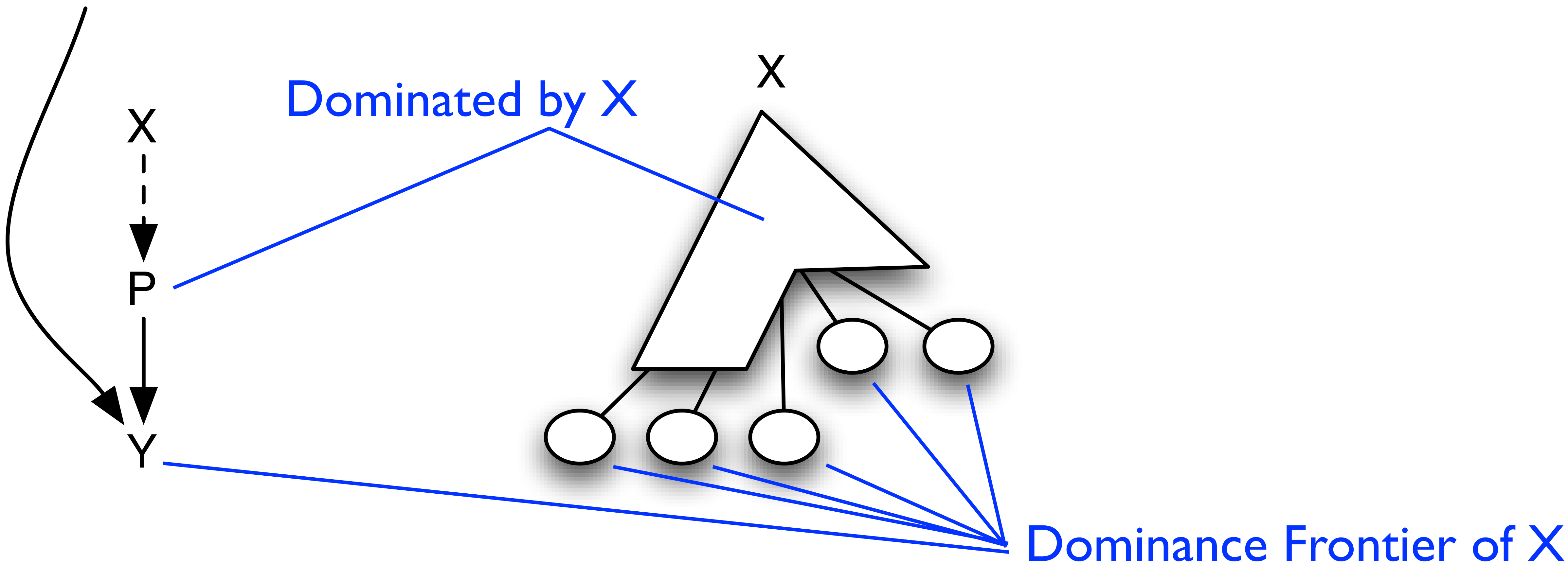
- Computing static single assignment form
- Computing control dependencies
- Identify loops in CFG
  - All nodes X dominated by entry node H, where X can reach H, and there is exactly one back edge (head dominates tail) in loop

Where do Φ Functions Go?

- We need a Φ function at node Z if
  - Two non-null CFG paths that both define v
  - Such that both paths start at two distinct nodes and end at Z



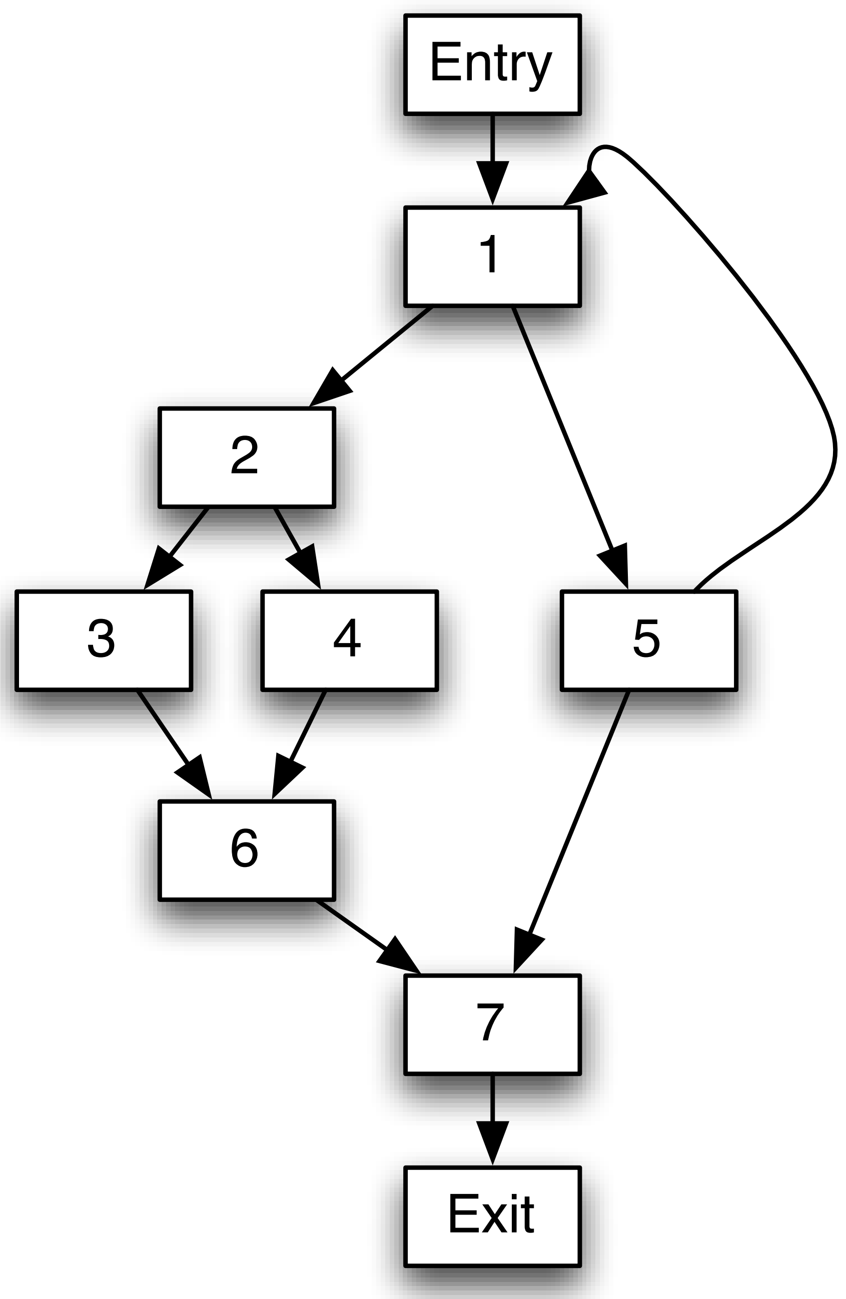
Dominance Frontiers: Illustration



Dominance Frontiers

- Y is in the dominance frontier of X iff
  - There exists a path from X to Exit through Y such that Y is the first node not strictly dominated by X
- Equivalently:
  - Y is the first node where a path from X to Exit and a path from Entry to Exit (not going through X) meet
- Equivalently:
  - X dominates a predecessor of Y
  - X does not strictly dominate Y

Example



$DF(1) = \{1\}$

$DF(2) = \{7\}$

$DF(3) = \{6\}$

$DF(4) = \{6\}$

$DF(5) = \{1, 7\}$

$DF(6) = \{7\}$

$DF(7) = \emptyset$

Computing SSA Form

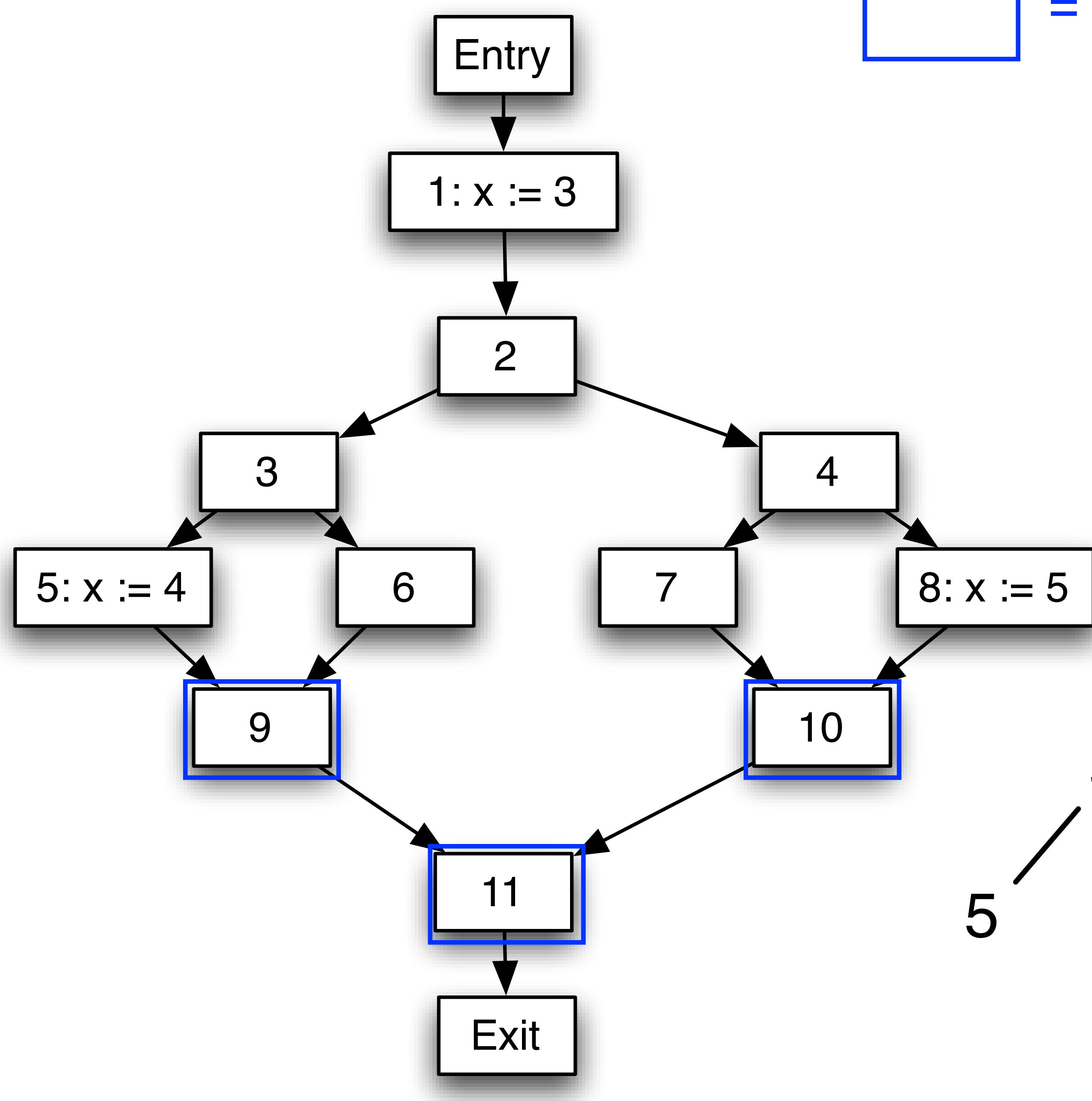
- Step 1: Compute the dominance frontier
- Step 2: Use dominance frontier to place Φ nodes
- Step 3: Rename variables so only one definition per name

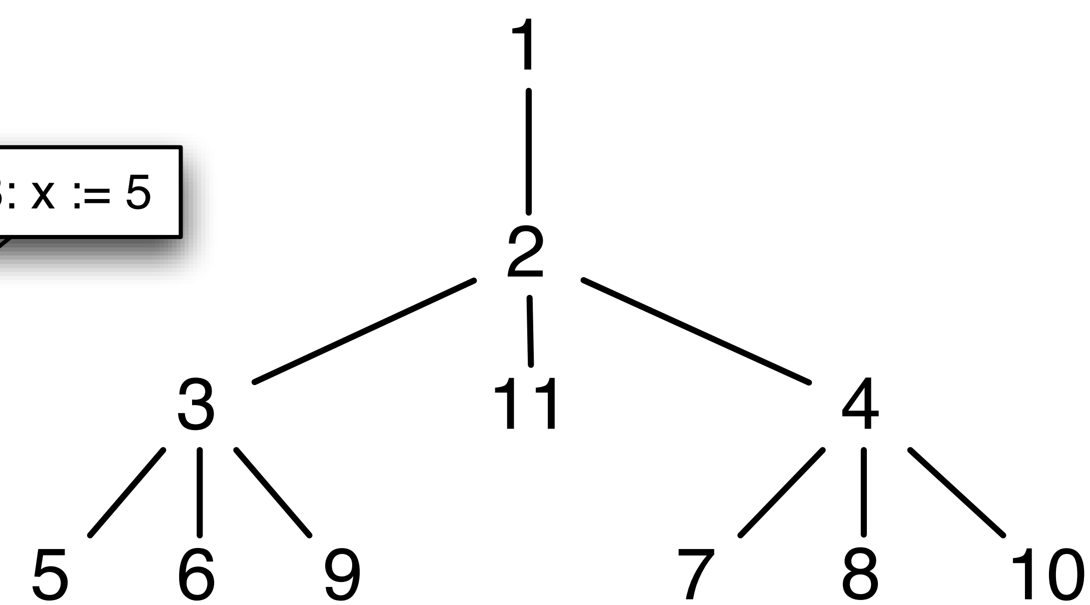
Step 2: Placing Φ Functions for v

- Let S be the set of nodes that define v
- Need to place Φ function in every node in DF(S)
  - Recall, those are all the places where the definition of v in S and some other definition of v may meet
- But a Φ function adds another definition of v!
  - v := Φ(v, ..., v)
- So, iterate
  - $DF_1 = DF(S)$
  - $DF_{i+1} = DF(S \cup DF_i)$

Example

= need Φ function





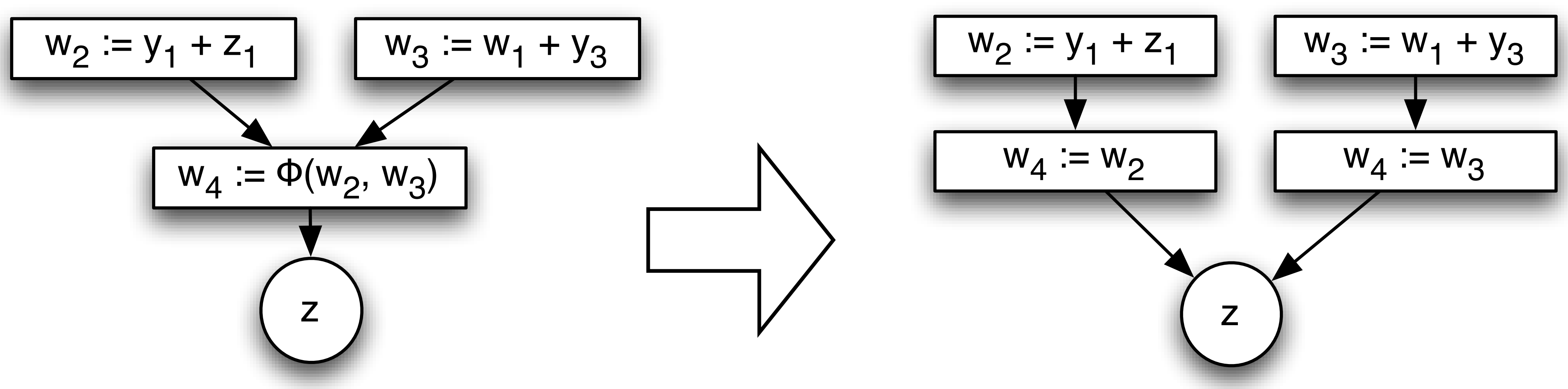
Step 3: Renaming Variables

- Top-down (DFS) traversal of dominator tree
  - At definition of v, push new # for v onto the stack
  - When leaving node with definition of v, pop stack
  - Intuitively: Works because there's a Φ function, hence a new definition of v, just beyond region dominated by definition

- Can be done in O(E+|DF|) time
  - Linear in size of CFG with Φ functions

Eliminating Φ Functions

- Basic idea: Φ represents facts that value of join may come from different paths
  - So just set along each possible path



Eliminating Φ Functions in Practice

- Copies performed at Φ fns may not be useful
  - Joined value may not be used later in the program
    - (So why leave it in?)

- Use dead code elimination to kill useless Φs

- Subsequent register allocation will map the (now very large) number of variables onto the actual set of machine register