Acquisition and development of monkey tool-use: behavioral and kinematic analyses[1]

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Abstract: Four Japanese macaques were trained in the use of a T-shaped rake. Use the tool and development of the level of the skill of tool-use took place in three distinct stages. During stage 1, two of the monkeys seemed to use insight for initial solution, while fortuitous experiences led the other two monkeys to the solution. All the monkeys used the tool in a stereotyped manner and could retrieve food only when the tool was placed close to the food. At stage 2 the monkeys became able to manipulate the tool in various ways and became able to retrieve the food regardless of its position. By stage 3 they had developed the level of skill required for efficient retrieval. Further experiments revealed that the monkeys attempted to use unfamiliar objects which were similar to the original tool in shape, but not spherical or ring-shaped objects, to rake in the food.

Key words: tool-use, macaque monkey, flexibility, biomechanics.

Résumé : Quatre macaques japonais ont été entraînés à utiliser un outil en forme de T. L'utilisation initiale de l'outil et le développement du niveau d'habileté nécessaire pour le manipuler se sont déroulés en trois étapes. Étape 1 : deux singes ont semblé avoir une compréhension intuitive de la solution initiale, alors que des expériences aléatoires ont dirigé les deux autres singes vers la solution. Tous les singes ont utilisé l'outil d'une manière stéréotypée et ont pu récupérer la nourriture seulement lorsque l'outil était placé près de celle-ci. Étape 2 : les singes ont appris à manipuler l'outil de diverses manières et à récupérer la nourriture peu importe sa position. Étape 3 : les singes ont développé le niveau d'habileté nécessaire pour une récupération efficace. D'autres expériences ont révélé que les singes ont essayé d'utiliser des objets non familiers dont la forme ressemblait à celle de l'outil initial, mais pas d'objets en forme d'anneau ou sphériques, pour récupérer la nourriture.

Mots clés : utilisation d'un outil, macaque, flexibilité, biomécanique.

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Introduction

Tool use, the employment of one object to alter the form, position, or condition of another object, can be seen in many species, ranging from birds to mammals (see reviews by Parker and Gibson 1977; Candland 1987;Tomasello and Call 1997). Among the various examples of tool-use, the stick problem, the use of a stick to rake in food that is placed out of reach of the animal, has been the most popular object of the study. In Köhler's early study (1927), chimpanzees picked up the stick after futile reaching for the food, then applied insight to rake in the food. Birch (1945) reported that playing with sticks was a prerequisite for tool-use in four out of six tested chimpanzees. Schiller (1952) presented a similar problem to chimpanzees but found that playing with sticks beforehand did not lead to improved performance on the problem. Thus, a consensus has not been reached regarding the solution of the stick problem by chimpanzees; according to Köhler the problem can be solved by insight, while according to Schiller the solution required trial-and-error based learning. Once the concept of the tool has been acquired, usage of the tool is not restricted to the original environment. For example, chimpanzees will break sticks off a shrub when an appropriate stick is not available, or will stack one box on another when a single box is not high enough for the chimpanzee to reach a banana hanging from a ceiling (Köhler 1927).

The use of sticks by macaque monkeys has also been reported upon, though to a lesser extent (Beck 1976; Iriki et al. 1996; Shurcliff et al. 1971; Tokida et al. 1994). Beck (1976) presented a problem which required the use of an L-shaped rake to a group of pigtailed macaques and found that initial solution by one group member resulted from trial-and-error, but subsequent solutions by other group members were accelerated by observational learning. The report by Tokida and his colleagues (1994) describes examples

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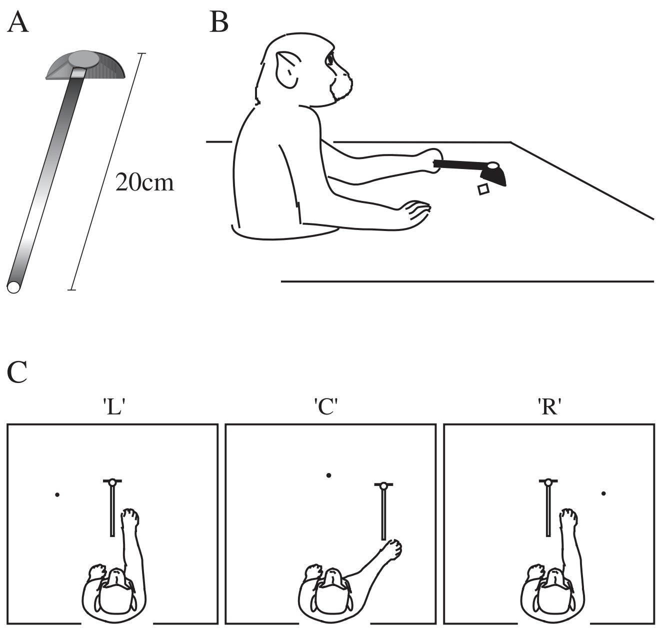


Fig. 1. Experimental setting. (A) The tool used for the training. (B) Drawing of the training schema: the rake-shaped tool was placed so that the monkey could use it to retrieve out-of-reach food. (C) The tool was placed at a distance laterally from the food in the test trials. Each panel shows the positions of the food and the tool in one of the test sets: left (L), center (C), and right (R).

of the solution of the food-in-pipe problem by a troop of free-ranging Japanese monkeys. When the experimenter first placed the L-shape tool within the pipe, six monkeys learned to use the tool to pull an apple from the pipe. Four of the monkeys then learned to insert the stick into the pipe to remove the food, after which they learned to throw stones to push the food out of the far end of the pipe. Since tool-use was examined in both the above studies within a group of monkeys, and as macaque monkeys have some capacity for observational learning (Beck 1976), the available literature on the solution of the out-of-reach problem without the use of observational learning seems still insufficient. Tool manufacturing by macaque monkeys was also described in Tokida et al. (1994). When only long sticks were available at the provisioning site, one Japanese monkey was seen to bite a long stick to shorten it. The same monkey was also once seen to pluck the leaves from a shrub, and bite a root to make a stick. Other than this example, there are few reports on the use of tools by macaques, unlike the case for chimpanzees, in environments that differ from the environment in which they were originally trained.

We aimed to: (*i*) identify how each Japanese monkey, in the absence of observational learning, starts to use the tool, as either by insight or by trial-and-error, (*ii*) analyze how each monkey develops skill, by introducing kinematic analyses in addition to the traditional observational approach, (*iii*) find parameters that can be used to quantify the skill, and (*iv*) examine whether macaque monkeys use the tool in the environment that differ from the environment in which they were originally trained.

Materials and methods

Four male Japanese monkeys (*Macaca fuscata*) weighing 3–5 kg were used. They were allowed water and monkey chow ad libitum. The animals were cared for according to the principles and guidelines of the Canadian Council on Animal Care. All experiments were conducted in an experimental chamber outside of the room in which the housing cages were kept, to avoid observational learning. Before being subjected to training, the animals were given acclimatization sessions until they showed no signs of agitation while sitting on the primate chair or of hesitating to extend their arms to get food from the table. The table was set to the monkey's waist height. The tool was kept on the table throughout this acclimatization period so that the monkeys could become visually familiar with it. However, it was placed out of reach to prevent the animals from playing with it. The tool used for the main part of this study was T-shaped, with an acrylic shaft 20-cm long and 1 cm in diameter, and a 6-cm wide semi-circular acrylic plate attached perpendicularly to the shaft as the tool's head. A marker was attached to the tool-head to allow kinematic analysis (Fig. 1A). Two other tools, each of which differed from the original one in color, length, and weight, were also used to test whether the monkeys could use unfamiliar tools that differ from the original T-shaped rake. This test was given to each monkey only once.

Training was scheduled for five days a week. In the afternoon on each training day, the monkeys were given 160–250 trials in five sessions. Each trial was initiated by placing a pellet of food on the table outside the monkey's reach and then placing the tool within the monkey's reach. The food reward was about one gram of apple or sweet potato in the form of 1-cm cube. The tool was placed with its shaft pointing towards the monkey (Fig. 1B) unless otherwise stated. The monkeys' behavior was considered to constitute a trial when the monkey came into contact with and manipulated the tool. A new trial was initiated after the monkey either obtained and finished eating the food, stopped trying, or when the food was displaced by the tool to an impossible distance to reach with the tool. Two types of trials, training trials and test trials, were given in a random order, but each type of trial constituted half of the trials on average. In the training trials, the positions of the food and the tool were determined according to the skill of the monkey. The training trials, namely, started in the way that is considered to offer the simplest condition for tool-use: the tool was placed just behind and in contact with the food, so that the food could be obtained by simply pulling the tool shaft (Fig. 1B). The distance between the food and the tool-head was kept within 5 cm for the first few days but was increased as training proceeded until day seven. The development of the monkeys' skill was assessed by the intermittent test trials. The positions of the food and tool in each test set were chosen from among three predetermined sets. All positions were 8 cm beyond the limit of the monkey's reach, with one position directly in front of the body. In each of the other two, there was an angle of 30 degrees between the direction of the food and the direction of the tool (Fig. 1C). In set L, the food and the tool were to the left and in the center position, respectively. In set C, they were respectively on the center and on the right mark, and these positions were reversed for set R. For the successful retrieval in any of the three test sets, it required movement of the monkey's forearm and elbow in a lateral direction [but not to pull elbow straight in (pull in) or rotate monkey's forearm at elbow (pull around) which were common throughout the training]. While sets L and R required monkeys to move the tool away from their bodies, the required motion for set C was in the direction of the monkeys. The success rate was calculated as the percentage of successful retrieval, defined by retrieval of the food in a single attempt for each test set or total of three test sets over the average of 29 neighbor test trials.

To allow evaluation of the monkeys' skill, all training was captured by a CCD color video camera (SONY, DXC-107, auto-

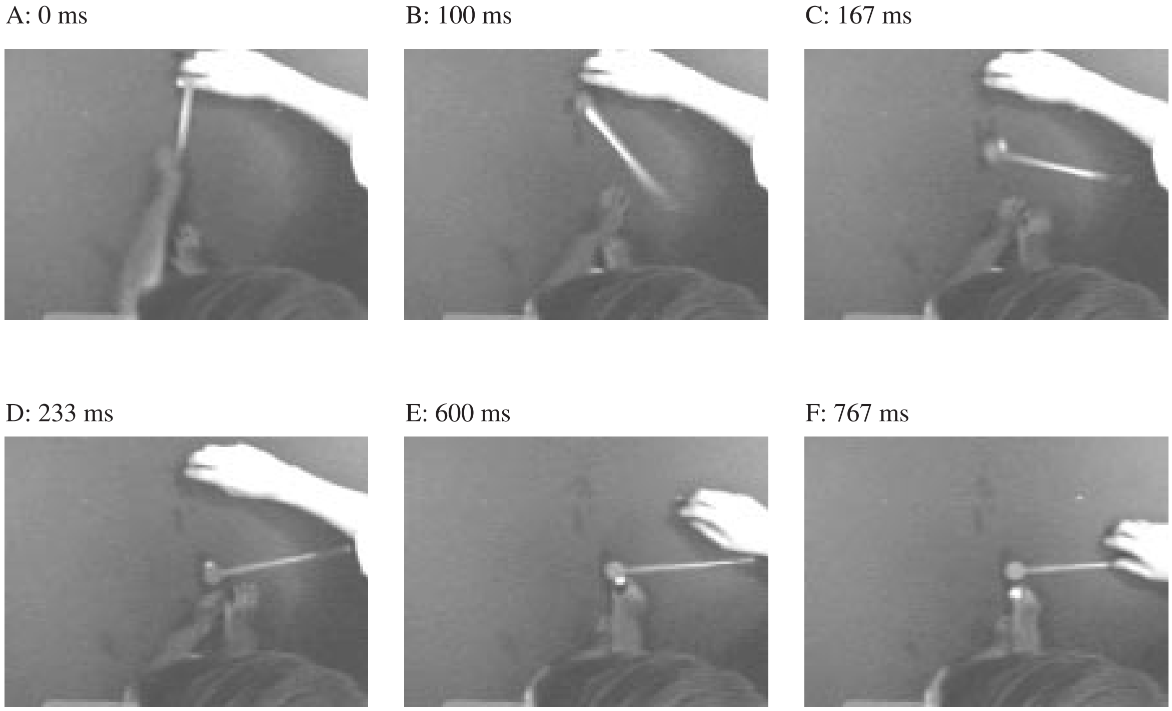


Fig. 2. An example of the fortuitous retrieval of the food. Training in tool-use was video-recorded through a video camera which was attached to the ceiling of the experimental room. The photographs are of the trial 40 s after the start of training on the first training day. The monkey, sitting on the bottom side of the figure, extended his arm to the tool shaft (A), then brushed it aside (B-C). The tool moved in a pivotal manner sweeping the food within reach of the monkey's hand (C and D). The monkey picked up the food (E and F). The times shown represent elapsed time from frame (A).

gain controlled). Captured images were sent to a video timer (FOR.A, VTG-33) for the addition of caption indicating the time of recording, and the captioned images were then sent to a digital videocassette recorder (SONY, DSR-30) and recorded for later analysis. A two-dimensional motion analysis system (Emtec, DOU-2020, resolution of 10 pixels/mm in real space) was used for kinematic analysis. The position of the tool-head was calculated as the center of gravity of the positive pixels in each frame at the video rate (30 frames/s), and this data was then sent to a personal computer (Apple, Power Macintosh) through a GPIB port under the control of specialized software (National Instruments, LabVIEW, Austin, Texas) and were then stored.

Results

Most processes underlying the development of tool-use appeared to be identical in principle for the various individuals. Thus, the description of behaviors and the kinematic analysis, other than the process of starting to be able to use the tool, deals with the results for a single monkey. In short, the acquisition and development of tool-use by these Japanese monkeys took place in three distinct stages: monkeys started to use the tool in a stereotyped manner (Stage 1), they acquired the skill to retrieve the food regardless of its placement (Stage 2), and then developed the level of skill required for efficient retrieval (Stage 3).

Stage 1: starting to use the tool and stereotyped behaviors

In the first few trials, when the tool was placed in contact with the food, all monkeys reached directly for the food and failed to obtain it. Two monkeys, after a few such attempts, grabbed the tool, pulled it, and obtained the food. The other two monkeys appeared to view the tool as a barrier or obstacle between their own bodies and the food, or at least did not see that it offered a positive functional value in the situation. These monkeys, after trying to reach the food directly the first few times, repeatedly brushed aside the tool's shaft and then tried again to reach the food. Although the brushing the tool aside usually resulted in the displacement of the tool's, but not the food's, position, it sometimes made the tool move in a pivotal manner and sweep the food along a short arc, and hence, to move the food within reach of the monkey's hand (Fig. 2). After experiencing this unexpected result once or twice, these two monkeys also started to hold and pull the tool shaft consistently to retrieve the food. Thus, all monkeys began to use the tool after a first few minutes, regardless of whether the first retrieval was intentional or resulted from random handling of the tool.

Until day seven, no remarkable change in tool-use behaviors was observed; instead, stereotyped behaviors, such as pull in or pull around to move the food towards their bodies dominated in this period. These behaviors were only successful in retrieving the food as long as the food lay between the tool and the monkey's body. When the food was placed at a distance laterally from the tool in the test trials (Fig. 1C), the monkeys either did nothing, pulled-in, or pulled-around, and never succeeded in retrieval. They did not make further attempts to retrieve the food, and usually released the tool, threw it away, or bit it.

Stage 2: emergence of variable actions and increased success rate

From around day seven, monkeys began to hold the tool for a longer time after a failure, and other new post-trial be-

Fig. 3. Representative paths of the tool-head in test set R of Stage 2. Each dot represents the position of the tool-head in a successive video frame (30 frames/s). Dots are interpolated. An asterisk in each figure represents the position of the food. Letters s and e represent positions of start and end. Figures are taken from training on days 12 (A) and 13 (B). (A) The tool was moved laterally to the food but overshot (single arrowhead), the monkey returned the tool to a pull-in motion (double arrowhead), which was followed by a lateral adjustment. (B) Lateral movement overshot (single arrowhead), but a single correction brought successful retrieval.

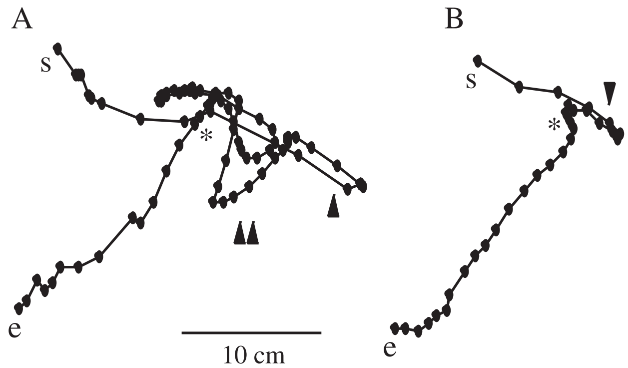
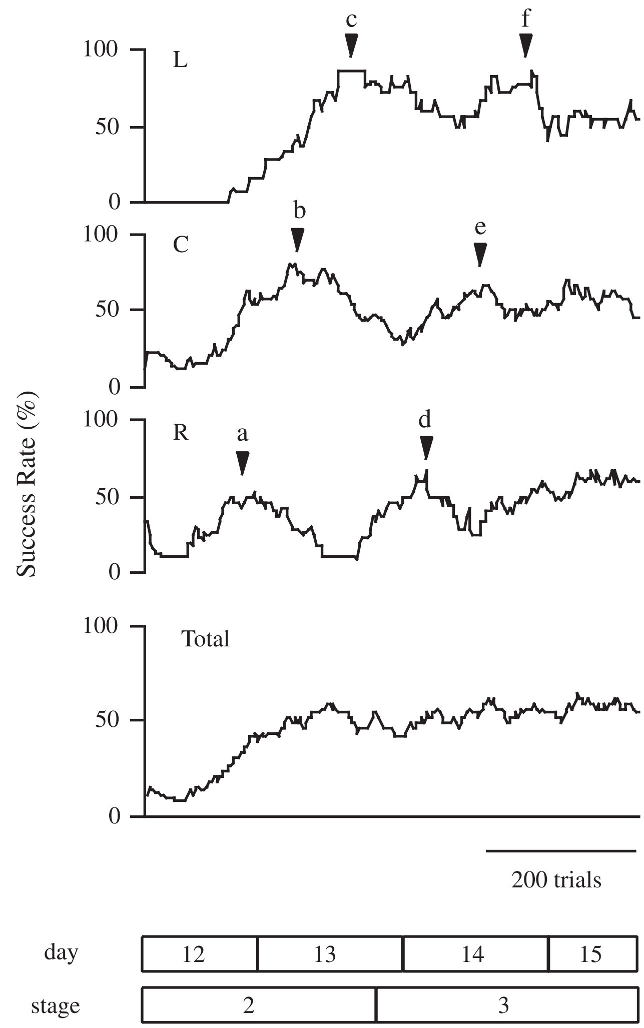


Fig. 4. Success rate for each test set, L, C, and R, and the total for all three sets (the lowest graph). Success was defined by retrieval of the food in a single attempt. Each arrowhead shows the peak performance over the global period of the test set.



haviors emerged. The monkeys began to move the tool in directions other than towards their own bodies, for example, pushing it or moving it laterally. Such behaviors were observed occasionally after they first emerged. Although these actions did not directly bring success in retrieval, they occasionally decreased the distance between the tool and the food. In some such cases, the monkeys initiated an additional attempt to get the food, i.e., a pulling-in action. These sequential actions, moving laterally and then pulling-in, marked the beginning of the combination of two previously independent actions, and appeared frequently from day 9 to 12. The strategy seemed to be consolidated over this period. The monkey then developed control of the lateral motion: the lateral motion often overshot or undershot and required correction on day 12 (Fig. 3A), but was more controlled on day 13 (Fig. 3B).

The rate of successful retrieval increased in accord with this development of the lateral motion. Figure 4 shows the success rate, as defined by success in retrieval on the first motion of the tool toward the monkey's body, in each test set and as a total of all three sets. Before day 12, distant food was rarely retrieved on the first attempt, but the performance on all over the three test sets improved from day 12 to 13 until the success rate had increased to 50%. The success rate then became stable (the bottom graph of Fig. 4). We have thus defined this period of stabilization of the lateral motion and an increase in the success rate as the end of Stage 2. During this time, the success rate rose and fell for each test set in turn (top 3 graphs of Fig. 4): the peak performance for set R was at the end of day 12 (arrowhead a), for set C at the beginning of day 13 (arrowhead b), and for set L in the middle of day 13 (arrowhead c). The same sequence of performance peaks appeared from day 13 to 14 (arrowheads d through f). This result suggests that the requisite learning was not common to the three sets and that only one pattern of tool-manipulation at a time could be learned effectively, and that single series of learning was insufficient for the development of the level of the skill.

An additional feature of this stage was an increase in willingness to continue the training concomitant to the improved performance. This was manifested as an extension of the time over which the training continued, and a decrease in the transfer of attention to environmental noise.

Stage 3: increased smoothness

The strategy of moving the tool laterally toward the food and then pulling it in continued to solidify until day 13. In most trials, the movements were observed to consist of two distinct segments, i.e., the monkey moved the tool laterally to the food, and then pulled it in (Fig. 5A). The typical velocity profile in the early phase of Stage 3 had two peaks with a wide trough between the peaks (solid line in Fig. 5B), showing that the two actions, moving laterally and pulling-in, were largely independently performed. The trough became narrower as the training proceeded into the middle phase of Stage 3 (dotted line in Fig. 5B), while the path remained much the same as for the earlier phase (Fig. 5A),

Fig. 5. Kinematic properties of tool-head in test set R over Stage 3. (A) Traces of the tool-head's path, chosen from early (solid line), middle (dotted line), and late phases (dashed line) of Stage 3. (B) The velocity profile of the tool-head for the three phases shown in (A). Time of impact on the food was set as time 0. (C) Velocity of the tool head on impact. Each dot represent the velocity of the tool head on impact in a single trial succeeded in retrieving the food. A second-order polynomial function is overlaid.

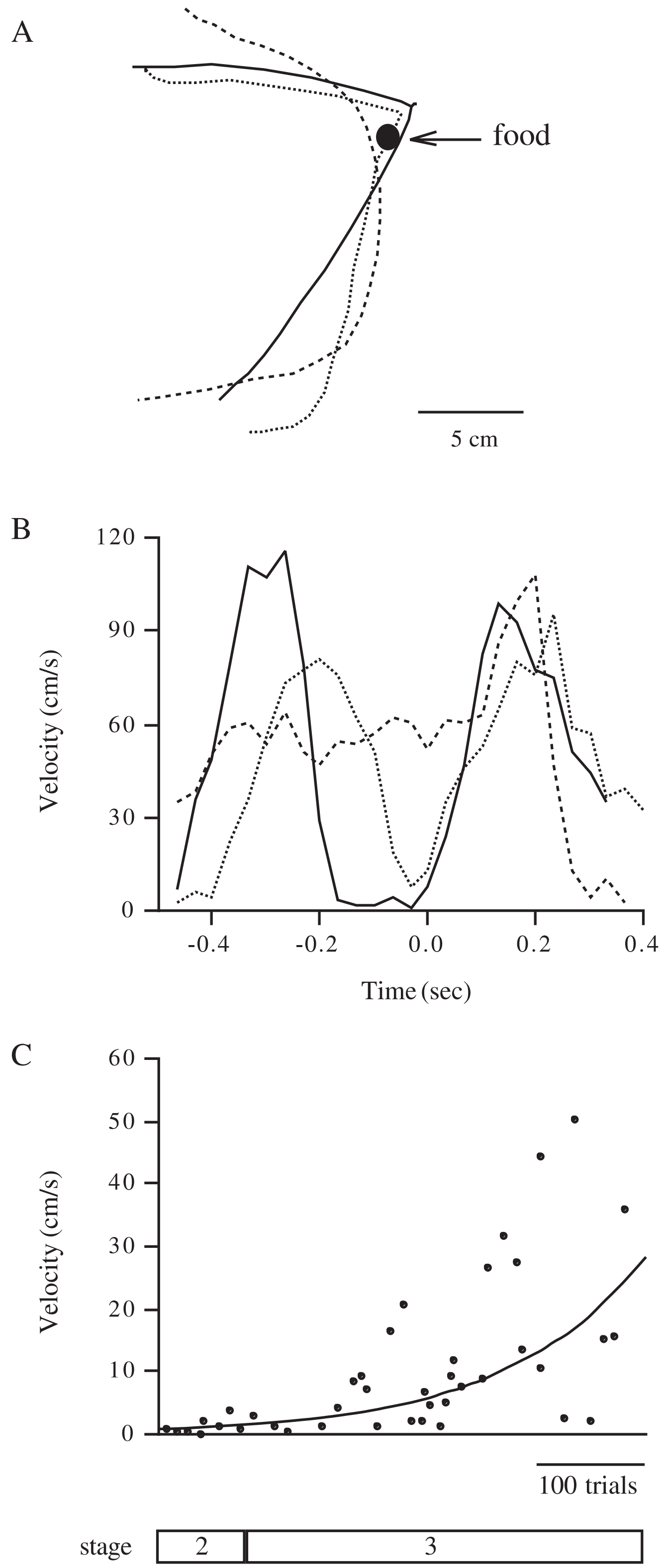
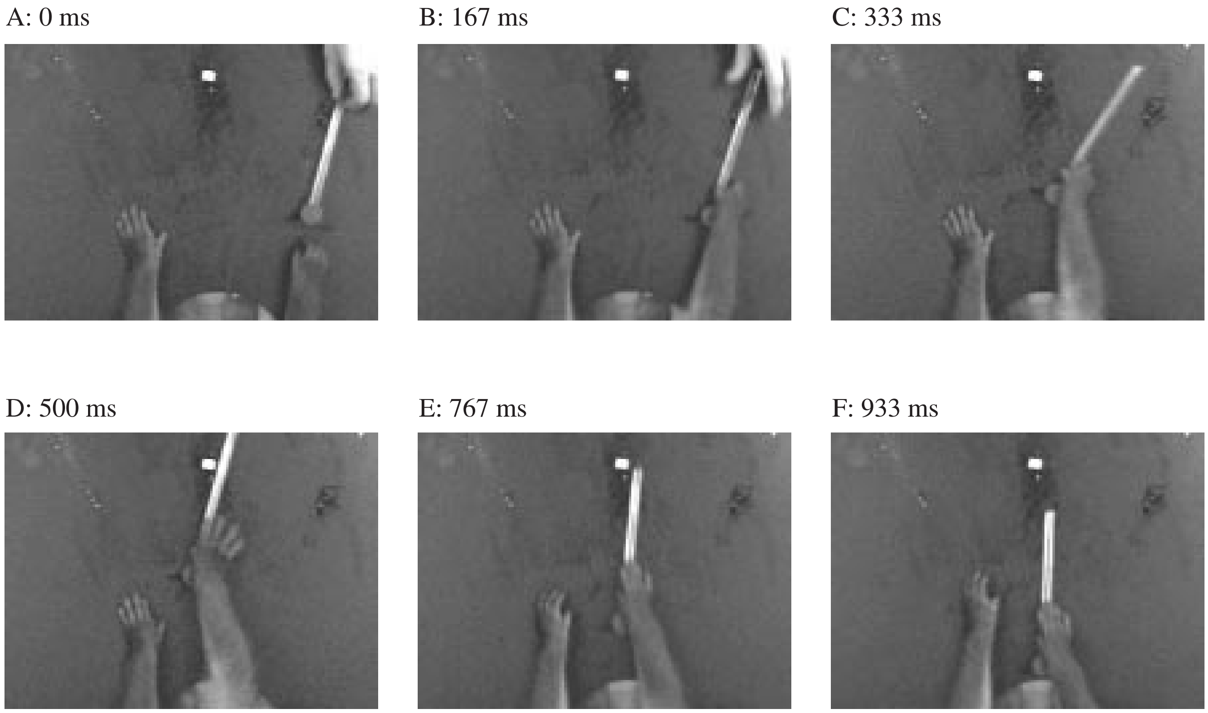


Fig. 6. The opposite-tool problem: the same conditions as for test set C, except with the tool in reverse orientation. The monkey held the tool as usual (A and B), moved it to the food (C and D), and pulled while reaching with the other hand (E and F). The monkeys stopped reaching for the food just after this and did not make any further attempts.



showing that the pause between the two actions had been shortened. Further training resulted in the tool being moved along a smooth path (dashed line in Fig. 5A) and the trough in the velocity profile disappearing (Fig. 5B), showing that the formerly lateral-and-pulling motions had now been integrated into a single smooth movement. The velocity of the tool-head as it hit the food, as shown in Fig. 5C, increased from the middle phase to the late phase of Stage 3. Some trials in late phase of Stage 3 still showed small velocity of the tool-head as it hit the food, in which cases the path had two segments. A possible reason is that although the path must be planned before starting to move the tool to be smoothly curved, the monkeys could not know the position of the tool beforehand since training trials and three test trials were randomly given.

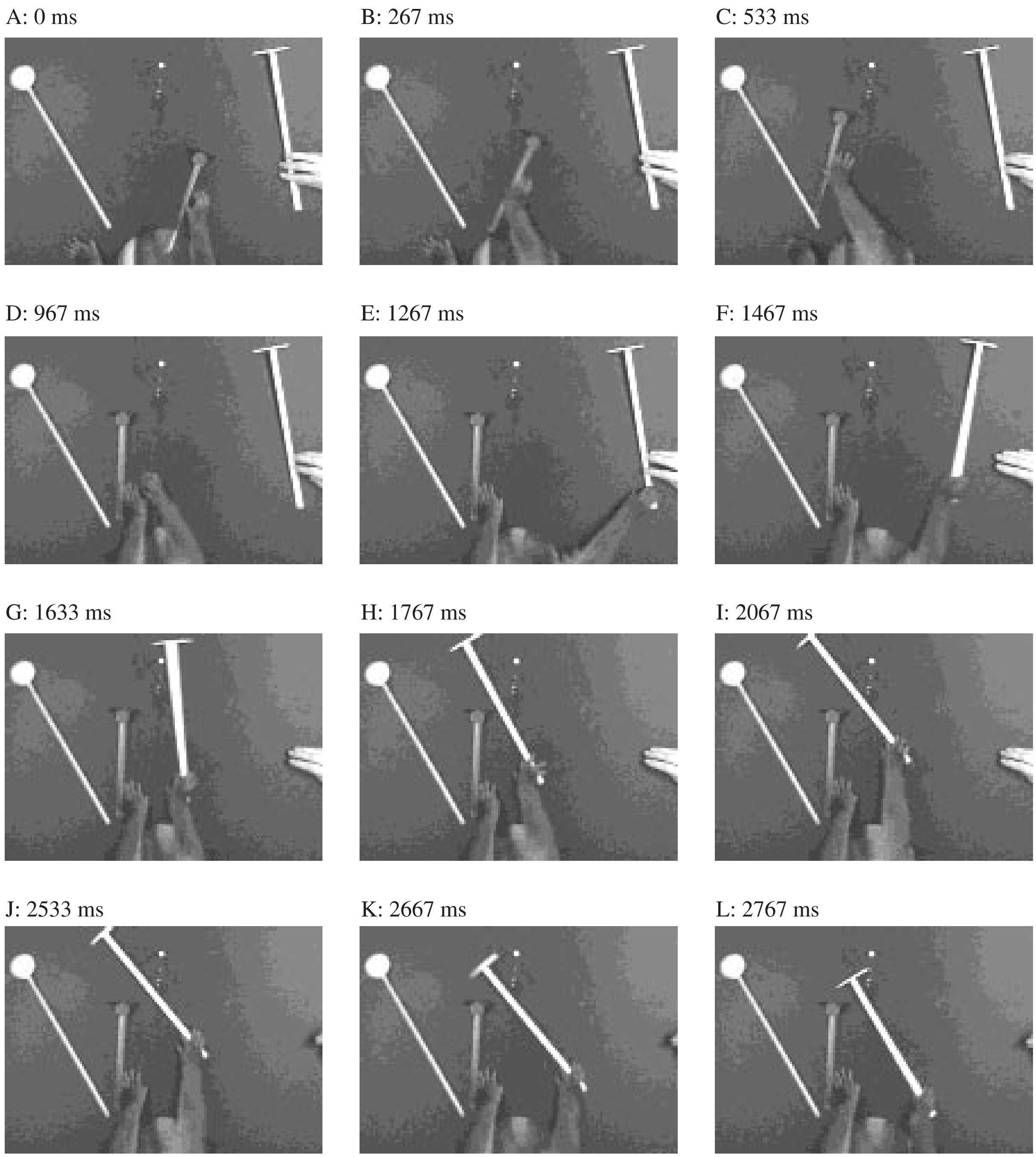
On some trials when the food was just outside of the reach of the tool, the monkeys released the tool once, then held the very end of the shaft, and tried to retrieve the food again. Monkeys did not make any attempt to retrieve the food when the food was placed further away, such that even the strategy of holding the very end of the shaft did not bring successful retrieval, suggesting that the monkeys came to know how far they could reach with the tool.

Comprehension of the rake as a tool

We included some experiments during the course of the training to examine the monkeys' comprehension of the rake as a tool. A few times every training day, we gave monkeys the tool in reverse orientation, i.e., with its head pointing toward the monkey. The monkeys rarely changed the orientation of the tool for efficient retrieval. Instead, in almost all cases they held the tool-head or the tool shaft close to the tool-head, moved the tool laterally, then pulled it in. Figure 6 shows a typical response to the opposite-tool problem, from day nine. The monkey started reaching toward the food while pulling-in (Fig. 6E and 6F). It took nearly one second before stopping the action. This raises a possibility that the monkeys did not conceptualize the function or shape or both of the tool. To test this possibility, we gave the monkeys a variety of toys which were spherical or ring-shaped. The monkeys typically twiddled with or bit them, but never tried to use them to retrieve the food, which shows that the monkeys conceptualized that not all objects could be used as the tool to rake in the food. We also tested the reaction of the monkeys to new tools. A monkey presented with a novel tool on day five first hesitated for a few seconds, then started to use it. Training with the new tool continued for five minutes. Another new tool was introduced and there was a further five minutes of training with this tool. The monkey again showed little hesitation in using the new tool. Manipulation of such novel tools was somewhat awkward, probably because they differed in length and weight from the original tool. We then gave the monkey all three tools, the original and two novel tools, to see which the monkey would use (Fig. 7). The monkey first took the most familiar one and manipulated it, then found that the tool was not long enough to reach the food. The monkey then released the first tool and took the white long tool and manipulated that. The results from these three experiments suggest that the monkeys conceptualized the stick-shape object as a tool to rake in the food.

In another experiment, we waved the food around instead of placing it on the table, and observed the behavior of the monkeys. The monkeys followed the waved food with the

Fig. 7. Selection and manipulation of familiar and novel tools. The monkey first took the familiar short tool that was placed nearest to him (A-C). It was not long enough to reach the food and was released (D). The monkey then took a novel but longer tool and moved it to the food (Fig. D-G), but overshot (H). He then pulled the tool (I-K), but failed to retrieve the food (Fig. L).

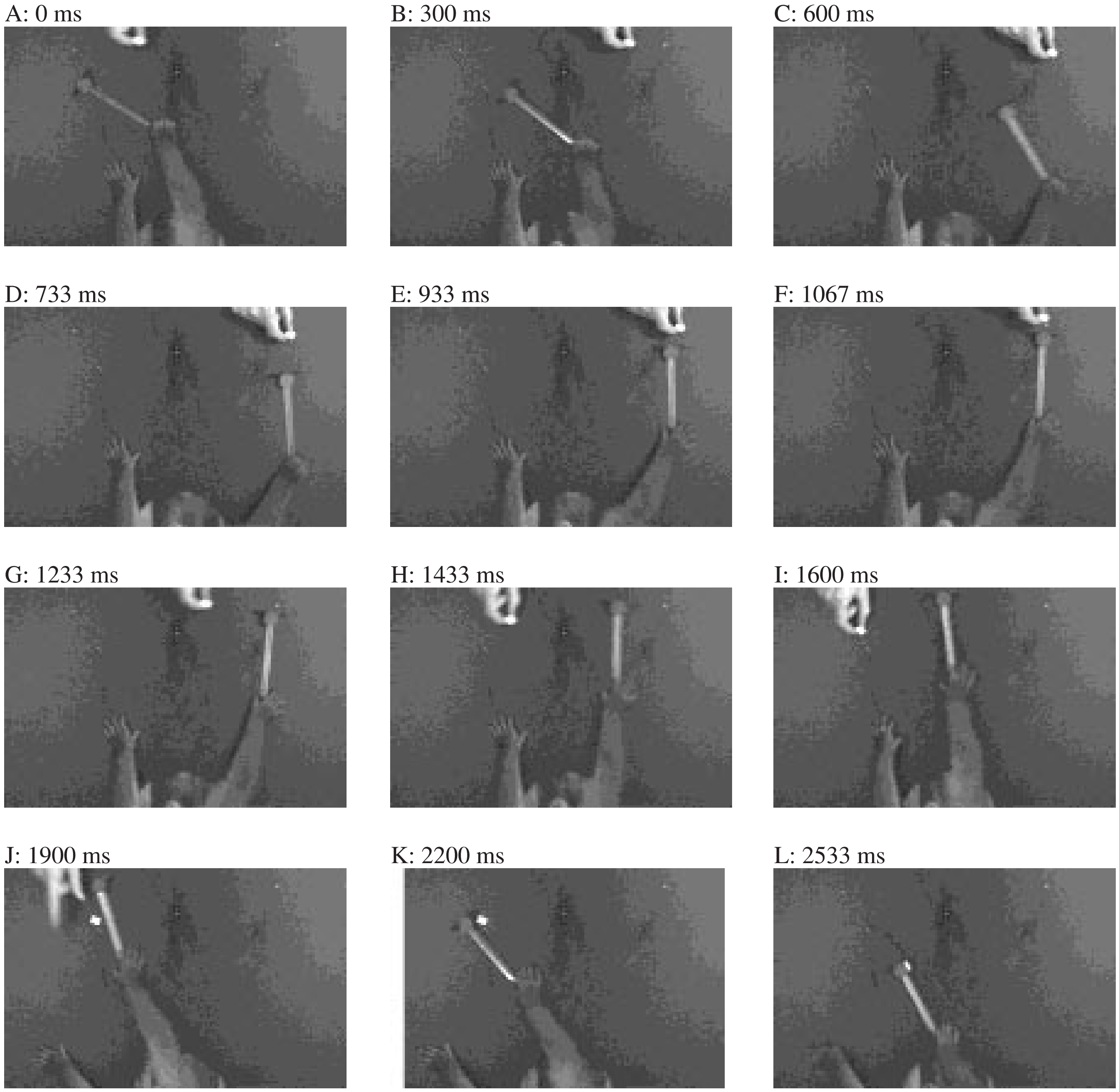


tool-head (Fig. 8), showing that the monkeys could not only retrieve food from a fixed position, but could also chase a moving target.

Discussion

No monkeys used the tool to retrieve the food on the first trial. To our surprise, all monkeys started to use the tool within the first few minutes. Two monkeys, after attempting to reach directly for the food a few times, grabbed the tool, pulled it in, and obtained the food. We contend that this was an intentional action and that the process shows insight, since the monkeys rarely failed to get the food by reaching for it alone. It is thus unlikely that they mistakenly came to hold the tool when they were in fact reaching for the food. The other two monkeys started to use the tool after fortuitously retrieving the food. This process cannot be considered as trial-and-error based learning. All four monkeys in our study thus showed clear difference in the initial solution of the problem from the monkeys in Beck's study (Beck 1976). Beck argued that initial acquisition by macaque monkeys was on a trial-and-error basis alone and that insight

Fig. 8. Using the tool to chase food with the tool-head. When the experimenter moved the food from left to right (A-C), the monkey chased the food with the tool-head (B-F). When the food was moved from right to left (G-I), the monkey again chased it with the tool-head (H-J) and retrieved (K and L).



was not involved. One reason for this discrepancy may result from the easier task which we set. In our task, the tool was placed just behind the food and thus only needed to be pulled in. In Beck's task, the tool was placed between the animal's cage and the food and the tool thus also needed to be pushed. Another possibility is that the tool was visually, though not conceptually, familiar to our monkeys before the training. Hence, our answer to the first question, how monkeys initiate tool-use, is that Japanese monkeys can initiate tool-use largely by insight if the conditions are appropriate.

Pushing and lateral movement of the tool in Stage 1 and at the beginning of Stage 2 were not goal-oriented and were unlikely to have been intentional since the motions frequently occurred while the monkeys were irritated or looking elsewhere, and they had no relation to position of the food. Why did it take so long for these Japanese monkeys to acquire the use of pushing and lateral motion? If one were to consider tool-use as an adaptation for feeding, as stated by Parker and Gibson (1977), the relatively slow emergence of pushing as compared with pulling can be explained in the following way. Pulling is a behavior which readily emerges in Japanese monkeys in their life in trees: a twig that carries fruit would commonly be grabbed and pulled to get the fruit, but pushing an inanimate object to retrieve food would rarely or never occur. The fact that the actions of pushing and lateral motion were acquired on a trial-and-error basis raises the possibility that the development of tool-use may merely be a consequence of conditioning or shaping and that the tool can be used only in the environment in which the monkeys were originally trained. Indeed, the monkeys manipulated the tool when it was placed in the opposite orientation as usual (Fig. 6), even though this way of manipulation is less efficient for successful food retrieval. However, we have evidence which show that these monkeys could use the tool in environments that differ from the environment in which they were originally trained. First, the monkeys used, without much hesitation, unfamiliar tools which were similar to the original tool in shape but differed from the original tool in such physical characteristics as color, length, and the weights of both shaft and head (Fig. 7). Second, when the experimenter waved the food around, the monkeys followed the food with the tool-head (Fig. 8).

The kinematic features of the retrieval of laterally placed food resemble those seen in the human via-point task (Uno et al. 1989). Uno et al. showed that the velocity profile became single-peaked when the via-point was located further away from the body than the line connecting the start and the target. The via-point in our experiments is the position of the food and thus is further from the monkey's body than the line connecting the start and end points. Thus, well-trained monkeys would be expected to show a single-peaked velocity profile, and we indeed observed it in the late phase of Stage 3, although it was not a clear bell shape. This may be because the kinematics we examined was that of an inanimate object, rather than of part of the monkey's own body. Another reason might be attributed to the difference of the geometry; the line connecting the start and end points was lateral in the experiments by Uno et al., while in ours it was in the general direction of the monkey's body.

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