Constrained Routing Between Non-Visible Vertices

Problem 1: Routing on Visibility Graph

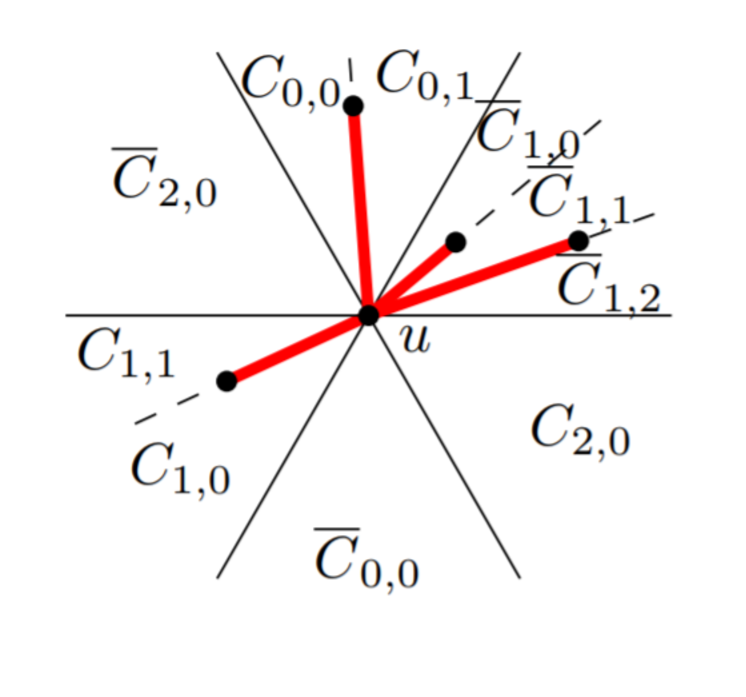
- We start with a set of vertices and constraints
- Trace all edges that don’t cross a constraint

- How can we efficiently route from a source vertex to a destination vertex on this graph?

The Θm-Graph

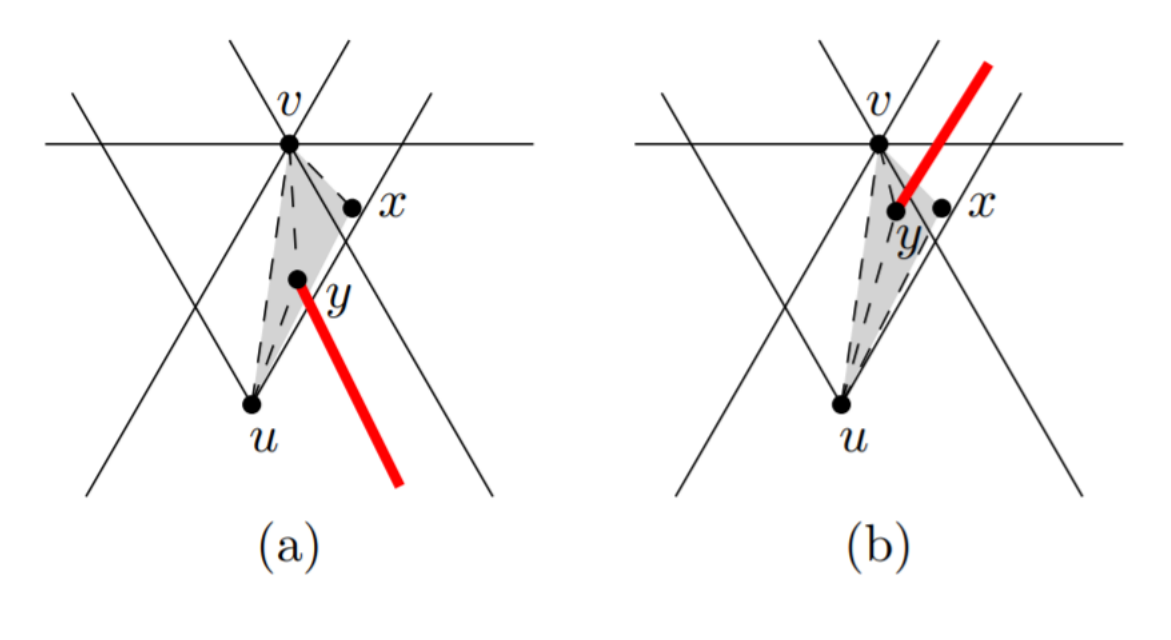
- Split the graph into m cones at each vertex.
- Trace an edge to the closest vertex in each cone.

- Half- Θm-Graph has positive and negative cones.



Problem 1: Routing on Visibility Graph

- Trace uv iff the projection of v to the bisector of the cone containing it is the closest among all vertices in the cone visible to v.
- Then we get a planar graph.
- Use routing techniques on planar graphs
- Result is deterministic and 1-local

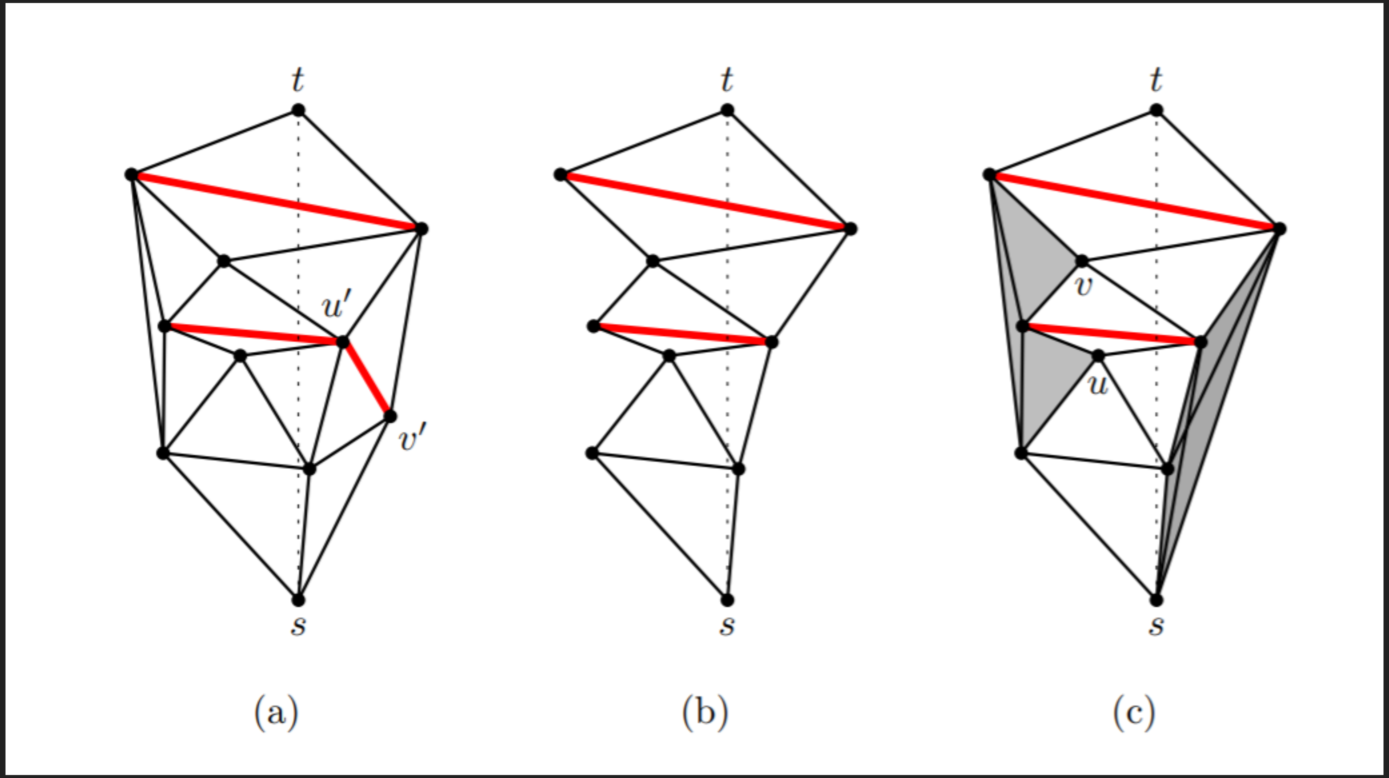


Problem 2: Routing on Constrained Triangulation

- Trace the segment from our source to the destination. Let H be the subgraph containing only the triangles intersected by this segment.
- We look at routing algorithms that focus only on H.

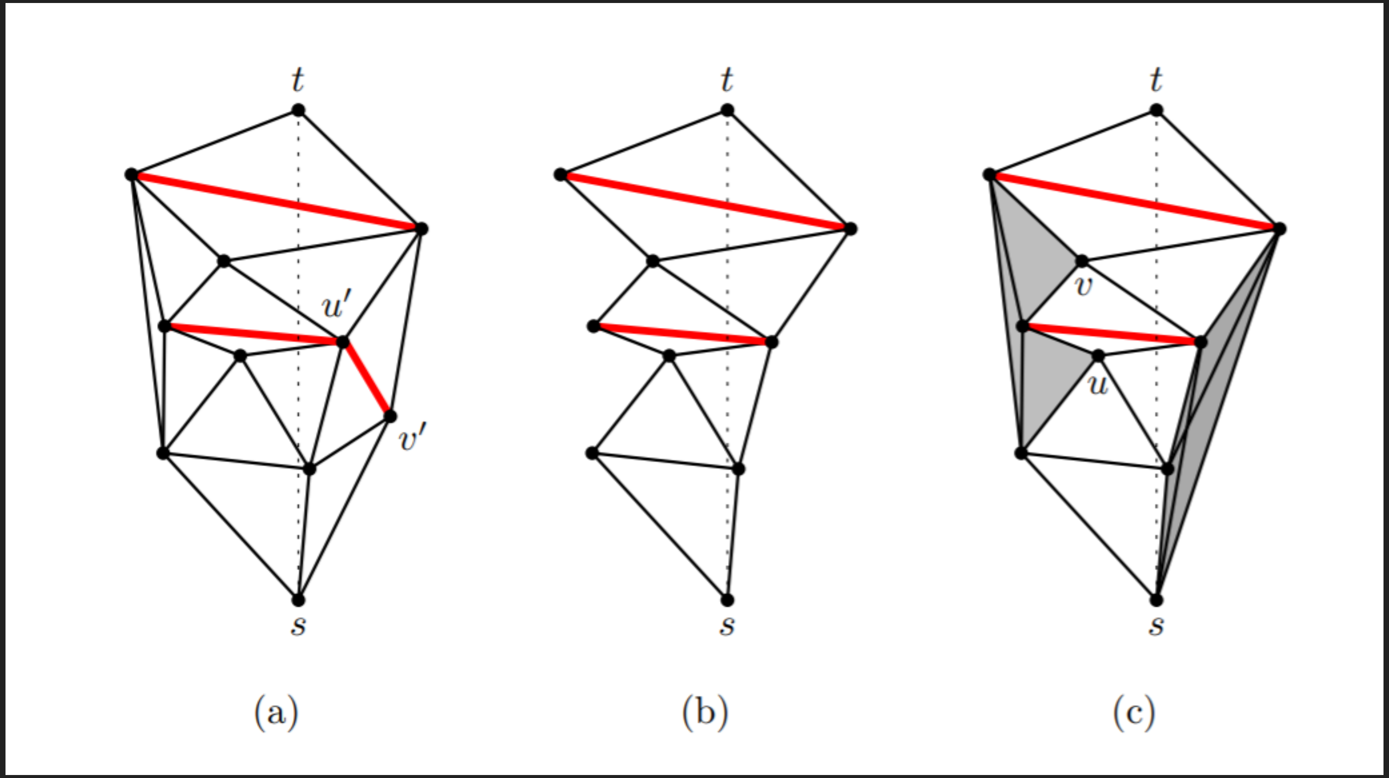
Problem 2: Routing on Constrained Triangulation

- Base efficiency on the ratio between the longest path In H and the longest path in G.
- Use a supplementary H’ (shown on the right)



Problem 2: Routing on Constrained Triangulation

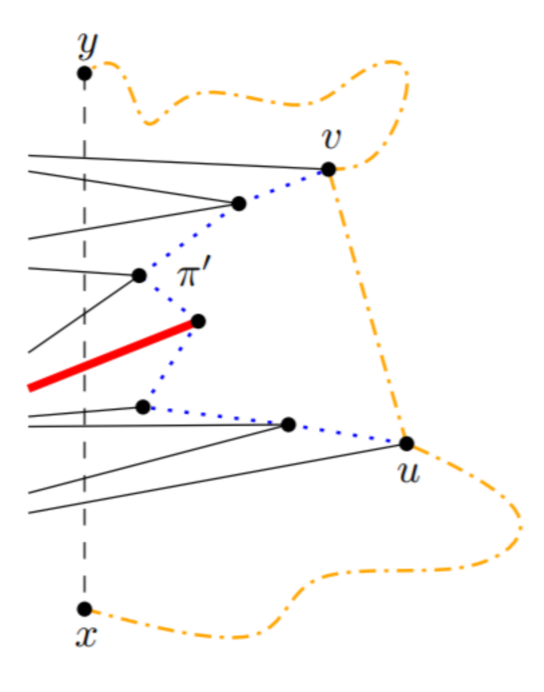
- The shortest path in H’ is no longer than the shortest path in G



(a) (b) (c)

Problem 2: Routing on Constrained Triangulation

- The shortest path in H is no longer than (n-1) times the shortest path in H'
- Idea of the proof:
  - Every edge in the shortest path of H can be replaced by a path of length no more than the size of this path in H
  - The path is contained in a polygon whose perimeter is at most twice the size of the shortest path in H'. The segment is then shorter than the shortest path in H'
  - Do this with all edges in the path in H' to get a path in H (there are at most (n-1) edges in the path in H')
- Shortest path in H ≤ (n-1) times shortest path in H' ≤ (n-1) times shortest path in G



Conclusions

- 1-local, deterministic algorithm to route in a visibility graph given some vertices and constraints.
- Provided bounds for routing algorithms on constrained triangulations that use the triangles intersected by the segment between the source and the target.