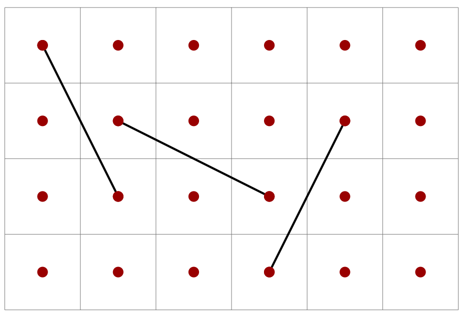
HW 10

Due: Fri, 14 Apr 2023

1. **Problem 7.2.8.** (!) On a chessboard, a **knight** can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate, as shown below. Prove that no $4 \times n$ chessboard has a **knight's tour**: a traversal by knight's moves that visits each square once and returns to the start. (Hint: Find an appropriate set of vertices in the corresponding graph to violate the necessary condition.)



2. **Problem 7.2.30.** Obtain Lemma 7.2.9 (sufficiency of Ore's condition) from Theorem 7.2.13 (sufficiency of Chvátal's condition). (Bondy [1978])

3. **Problem 7.2.31.** (!) Prove or disprove: If G is a simple graph with at least three vertices, and G has at least $\alpha(G)$ vertices of degree $n(G) - 1$, then G is Hamiltonian.

4. **Problem 3.1.8.** (!) Prove or disprove: Every tree has at most one perfect matching.

5. **Problem 3.1.9.** (!) Prove that every maximal matching in a graph G has at least $\alpha'(G)/2$ edges.