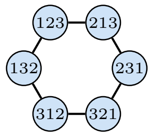
HW 2

Due: Fri, 3 Feb 2023

Key from text: (−) = easier (+) = harder (!) = useful or instructive (*) = uses optional material

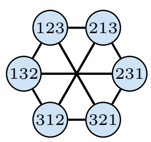
1. **Problem 1.2.17.** (!) Let G_n be the graph whose vertices are the permutations of $(1, \ldots, n)$, with two permutations $a_1, \ldots, a_n$, and $b_1, \ldots, b_n$, adjacent if they differ by interchanging a pair of adjacent entries (G_3 shown below). Prove that G is connected.



Solution: . . . .
Your solution here. ♠♡♣♢

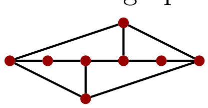
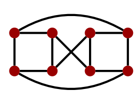
2. **Problem 1.2.20.** Let v be a cut-vertex of a simple graph G. Prove that $\overline{G} - v$ is connected.

3. **Problem Problem 1.2.27.** Let G_n be the graph whose vertices are the permutations of $(1, \ldots, n)$ with two permutations $(a_1, \ldots, a_n)$ and $(b_1, ..., b_n)$ adjacent if they differ by switching two entries. Prove that G_n is bipartite.



4. **Problem 1.2.28.** (!) In each graph below, find a bipartite subgraph with the maximum number of edges. Prove that this is the maximum, and determine whether this is the only bipartite subgraph with this many edges.

Use the theory, not brute force.

5. **Problem 1.2.29.** (!) Let G be a connected simple graph not having P_4 or C_3 as an induced subgraph. Prove that G is a biclique (complete bipartite graph).

__induced__ subgraph defined on p.23

6. **Problem 1.2.40.** Let P and Q be paths of maximum length in a connected graph G. Prove that P and Q have a common vertex.

Optional bonus question. Let P be the Petersen graph.

The standard proof of (c) uses eigenvalues, but you can do in an "elementary", but slightly tedious, way. Use the Kneser labeling.

(a) Find a copy of C_{10} in the complement of P.
(b) Is there a copy of P in the complement of P?
(c) Are there two edge-disjoint copies of P in the complement of P?
N.B. Just counting edges tells us there are at most 2 such edge-disjoint copies of P.