HW 7

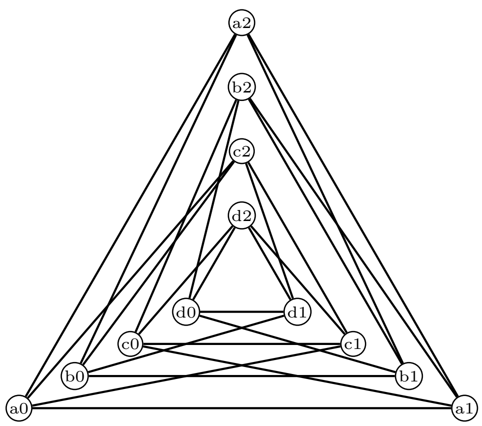
Due: Fri, 17 Mar 2023

1. **Problem 6.1.6.** (–) Prove that a plane graph is 2-connected if and only if for every face, the bounding walk is a cycle.

Definition. A graph is **outerplanar** if it has an embedding with every vertex on the boundary of the unbounded face.

2. **Problem 6.1.7.** (–) A **maximal outerplanar graph** is a simple outerplanar graph that is not a spanning subgraph of a larger simple outerplanar graph. Let G be a maximal outerplanar graph with at least three vertices. Prove that G is 2-connected.

3. **Problem 6.1.13.** Find a planar embedding of the graph below. *Note: You must explain how you got your planar embedding. The text gives an answer without explanation... My answer does not look like the text answer, but I give an explanation of how I got it.*



- **Problem 6.1.21.** (!) Prove that a set of edges in a connected plane graph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*. *Don't turn in, but take a look at this one. We didn't talk about spanning trees in class, but this shows a nice connection between duals and spanning trees.*

4. **Problem 6.1.25.** (!) Prove that every n-vertex plane graph isomorphic to its dual has $2n - 2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

Bonus problem ....

5. Let G be connected planar. Find a condition on G that ensures G^* is simple.