HW

Due: Fri, 31 Mar 2023

Not graded. Do not turn in.

(But there still might be a quiz on Wed, Apr 5)

1. **Problem 6.2.2.** (−) Give three proofs that the Petersen graph is nonplanar.

   (a) Using Kuratowski's Theorem.

   (b) Using Euler's Formula and the fact that the Petersen graph has girth 5.

   (c) Using the planarity-testing algorithm of Demoucron-Malgrange-Pertuiset.

   Skip this part, but you might be interested in taking a look in the text ...

2. **Problem 6.2.6.** (!) Fáry's Theorem. Let R be a region in the plane bounded by a simple polygon with at most five sides (simple polygon means the edges are line segments that do not cross). Prove there is a point x inside R that "sees" all of R, meaning that the segment from x to any point of R does not cross the boundary of R. Use this to prove inductively that every simple planar graph has a straight-line embedding.

3. **Problem 6.3.1.** (−) State a polynomial-time algorithm that takes an arbitrary planar graph as input and produces a proper 5-coloring of the graph.

4. **Problem 6.3.3.** (−) Use the Four Color Theorem to prove that every outerplanar graph is 3-colorable.

5. **Problem 6.3.5.** Prove that every planar graph decomposes into two bipartite graphs. (Hedetniemi [1969], Mabry [1995])
Hint: Use the 4*-color Theorem.*