COMP 150-SEN
Software Engineering Foundations

Program Verification (cont'd)

Spring 2019

Review: Verification

- Given: A program and a specification
- Goal: Prove the program abides by the spec
- Unlike testing, verification can show that a program is correct for *all* inputs
  - Testing: assert(abs(-5) >= 0), assert(abs(10) >= 0), assert…
  - Verification: assert(∀ int x : abs(x) >= 0)

Review: Weakest Preconditions

- Given: a program statement `S` and a postcondition `B`
- Find: weakest precondition `A` for `S` such that `B` will hold
  - `A` is *weaker* than `A’` if `A` is true in more states
    - E.g., `x > 0` is weaker than `x > 5`
    - Thus, `A` is weaker than `A’` if `A’ ⇒ A`
    - (`true` is the weakest possible assertion)
  - Thus, the weakest precondition `A` means
    - `{A} S {B}` is valid (`A` is a precondition)
    - If `{A’} S {B}` is valid, then then `A’ ⇒ A`

Review: Weakest Precondtions

- Define `wp(S, B)` to be the weakest precondition of `B`, as follows
  - wp(x=e, B) = B[x↦ e]
  - wp(S1; S2, B) = wp(S1, wp(S2, B))
  - wp(if E then S1 else S2, B) =

    $E \Rightarrow wp(S1, B) \wedge \neg E \Rightarrow wp(S2, B)$

Loops

- There is no mechanical way to automatically check whether loops are correct
- Two key problems
  - Weakest preconditions might not terminate, because it needs to keep going backward through the loop repeatedly
  - We'd like to also know whether loops terminate, which weakest preconditions doesn't reason about

- Instead, we're going to develop a pen-and-paper way of reasoning about loops

Summation in a Loop

- Goal: prove the following triple correct

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

  - Notice this is total correctness, i.e., we need to prove the loop terminates
  - (We will ignore concerns about the array length)

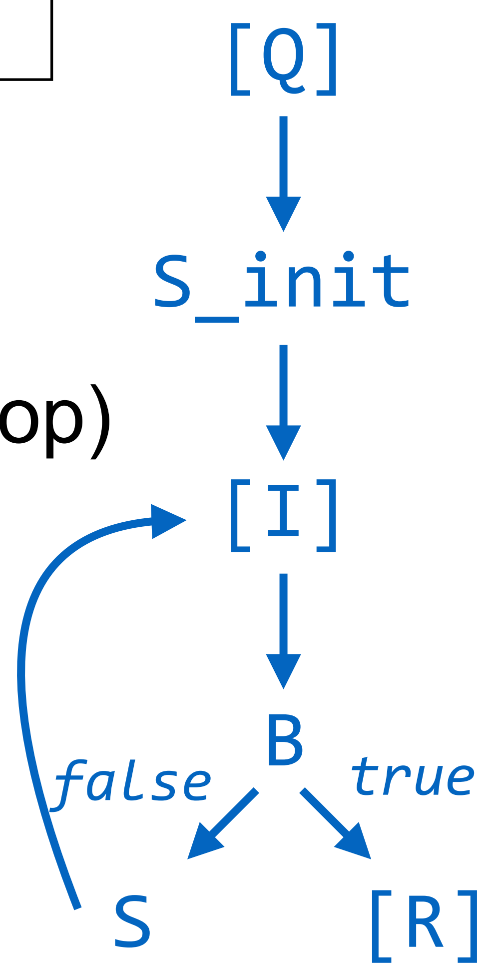
Loop Invariants

- Key idea: think about one loop iteration, generically
- A *loop invariant* is an assertion that holds at the beginning and end of each execution of the loop

```
[Q] S_init; while(B) { S } [R]
```

  - Let `I` be the loop invariant

- To prove this loop correct
  1. Show `[Q] S_init [I]` (invariant holds before loop)
  2. Show `[I∧B] S [I]` (invariant holds across loop)
  3. Show `I∧¬B⇒ R` (post holds after loop)



Loop Invariant Example

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
     0 ≤ k < n)
```

1. `[true] sum=a[0]; k=0; [sum=a[0]+…+a[k] ∧ 0≤k<n]`
   - Holds by sequence and assignment wp rules

Loop Invariant Example (cont'd)

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
    0 ≤ k < n)
```

2. `[sum=a[0]+…+a[k] ∧ 0≤k<n ∧ k ≠ n-1]`

   `k=k+1; sum=sum+a[k];`

   `[sum=a[0]+…+a[k] ∧ 0≤k<n]`

   - Holds by assignment rules and consequence

Loop Invariant Example (cont'd)

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
I: (sum = a[0]+…+a[k] ∧
    0 ≤ k < n)
```

3. (sum=a[0]+…+a[k] ∧ 0≤k<n ∧ k = n-1) ⇒

   sum = a[0]+a[1]+…+a[n-1]

   - Holds by standard logical reasoning

Bound Function for Termination

- A *bound function* `t` is
  - An integer-valued expression defined in terms of program variables
  - `t` must strictly decrease with every loop iteration
  - `t≥0` always, so that when it reaches `0` the loop terminates

- I.e., a bound function is an upper bound on the number of remaining loop iterations

Bound Function Example

```
[ true ]
sum = a[0];
k = 0;
while (k ≠ n-1) {
  k = k  + 1;
  sum= sum + a[k];
}
[ sum =  a[0]+a[1]+…+a[n-1] ]
```

```
t:  n-1-k
```

- Check that `t` is a bound function
  - `t` strictly decreases with each step because `k` increases by `1`
  - `t` can never go below zero because the loop terminates with `k=n-1`
- Therefore, the loop terminates

Verification Conditions

- A *verification condition* (VC) is a logical formula that is valid if a program abides by its spec
  - Formula *validity* means it is true for any assignment to its variables
- We can verify programs if we have ways to:

1. Generate VCs based on programs and their specs.
2. Check the validity of those VCs.

- For (1), we can use weakest preconditions!
- Given: Hoare triple `{A} S {B}`
- We can generate VC: `A ⇒ wp(S, B)`

Generating VCs

- Given program:
  `if x≥0 then abs_val = x else abs_val = -x`
- Given postcondition: `abs_val≥0`
- No precondition (i.e., precondition is just `true`)
- Goal: use weakest preconditions to generate VC

Using Weakest Preconditions

- Recall wp rules for assignment and conditionals
  - wp(x=e, B) = B[x↦ e]
  - wp(if E then S1 else S2, B) =

    E⇒ wp(S1, B) ∧ ¬E⇒ wp(S2, B)

```
wp(if x≥0 then abs_val=x else abs_val=-x, abs_val≥0)
```

Apply conditional rule:

```
= (x≥0) ⇒ wp(abs_val=x, abs_val≥0) ∧
  ¬ (x≥0) ⇒ wp(abs_val=-x, abs_val≥0)
```

Apply assignment rule:

```
 = (x≥0) ⇒ (x≥0) ∧ ¬ (x≥0) ⇒ (-x≥0)
```

Generating, Checking VCs

- The weakest precondition was:

$$(x \geq 0) \Rightarrow (x \geq 0) \land \neg (x \geq 0) \Rightarrow (-x \geq 0)$$

- Because the precondition of the program was `true`, the VC for this program is:

$$\text{true} \Rightarrow ((x \geq 0) \Rightarrow (x \geq 0) \land \neg (x \geq 0) \Rightarrow (-x \geq 0))$$

- Now, must check validity of VC
- Could write handwritten proof, but an automated proof is preferable

SMT Solvers

- *Satisfiability modulo theories (SMT)*
- SMT solvers extend boolean satisfiability solving with theories from other domains
  - Satisfiability: Is there an assignment to the variables of a formula that makes the formula true?
  - Integers, real numbers, arrays, records, …
- Allow us to check the satisfiability of formulas involving more than just boolean logic

```
true ⇒ ((x≥0) ⇒ (x≥0) ∧ ¬ (x≥0) ⇒ (-x≥0))
```

- Recent (~15 years) progress in SMT solving is a *major* reason for advances in program verification

Using SMT Solvers

- Z3 is a popular, state-of-the-art solver
  - Developed at Microsoft Research by Leonardo de Moura, Nikolaj Bjorner
- We will use it to check if our VC is valid
- A formula is valid if and only if its *negation is unsatisfiable*
- So, we will check the satisfiability of VC's negation:

$$\neg\ (\text{true} \Rightarrow ((x \geq 0) \Rightarrow (x \geq 0) \land \neg (x \geq 0) \Rightarrow (-x \geq 0)))$$

- Here it is in Z3: https://rise4fun.com/Z3/4slBH

Putting it all Together

- We have a way to generate VCs
  - Using weakest preconditions
  - Not mechanical for loops, though: need manually provided loop invariants
- We have a way to check the validity of VCs
  - Using SMT solvers
- Putting these together, we have a way to achieve program verification

Dafny

- Dafny is an object-oriented, compiled language with constructs for verification
- Supports formal specs through preconditions, postconditions, and loop invariants
- All specs are verified
  - VCs generated with weakest preconditions, checked with Z3
  - Many, many more algorithms and heuristics used
  - Dafny also proves program termination (w/ programmer help)
- Also developed at Microsoft Research, under direction of Rustan Leino

Using Dafny

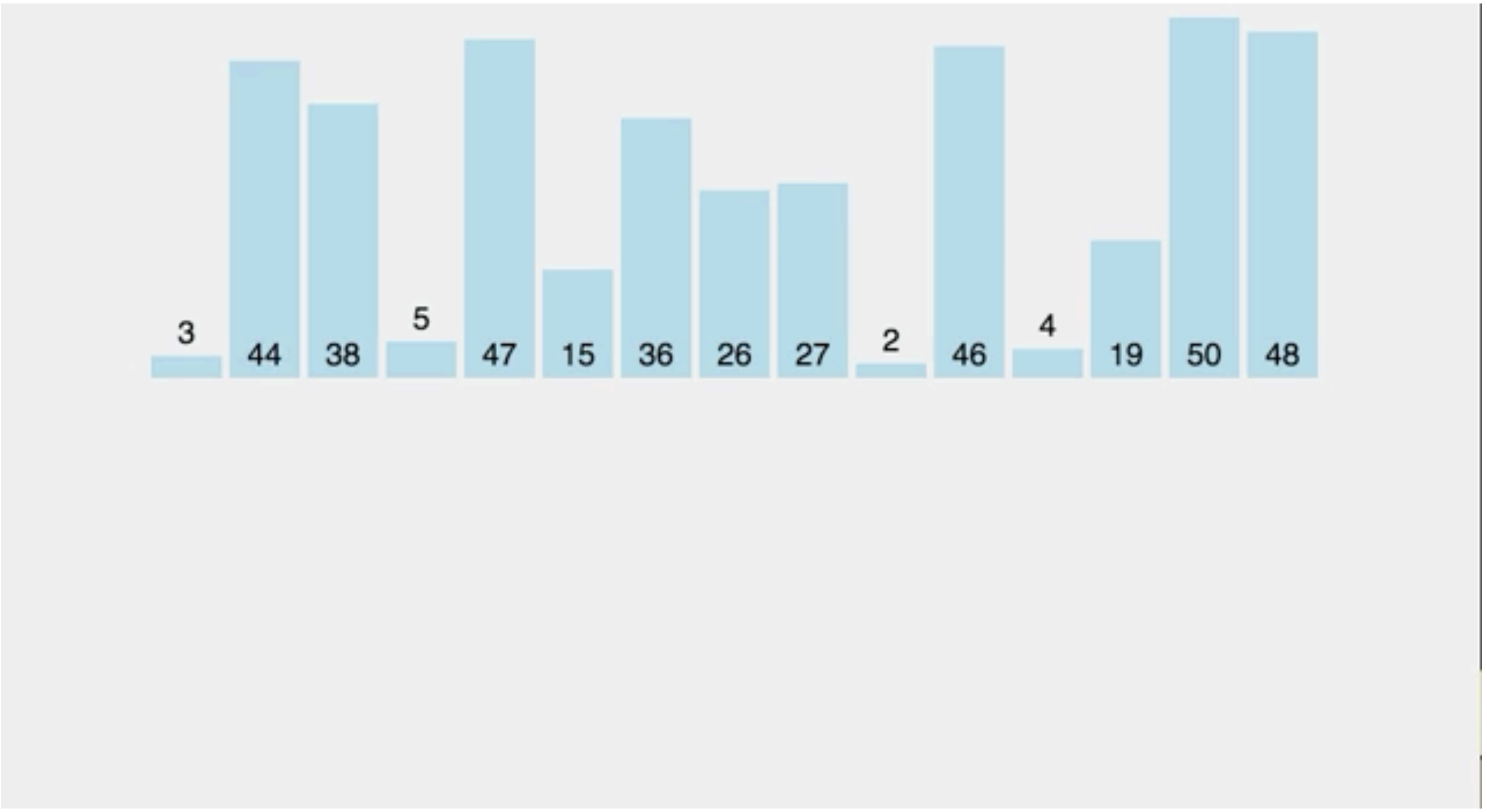
- Biggest challenge is coming up with loop invariants
  - No automatic procedure for doing so
  - But, there are heuristics we can use to make good guesses

- We will use Dafny to verify absolute value method, and insertion sort

Review: Insertion Sort

- Pseudocode:

```
i ← 1
while i < length(A)
    j ← i
    while j > 0 and A[j-1] > A[j]
        swap A[j] and A[j-1]
        j ← j - 1
    end while
    i ← i + 1
end while
```



Using Dafny

- Demo: https://rise4fun.com/Dafny/op05p

- Fully annotated version: https://rise4fun.com/Dafny/GxFe

Using Dafny

- Verifying specs is reasonably practical!
- Some aspects, especially loops, are tricky
- But, it shows the promise of new verification technologies
- Many more verification “success stories” in recent years…

Ironclad Apps

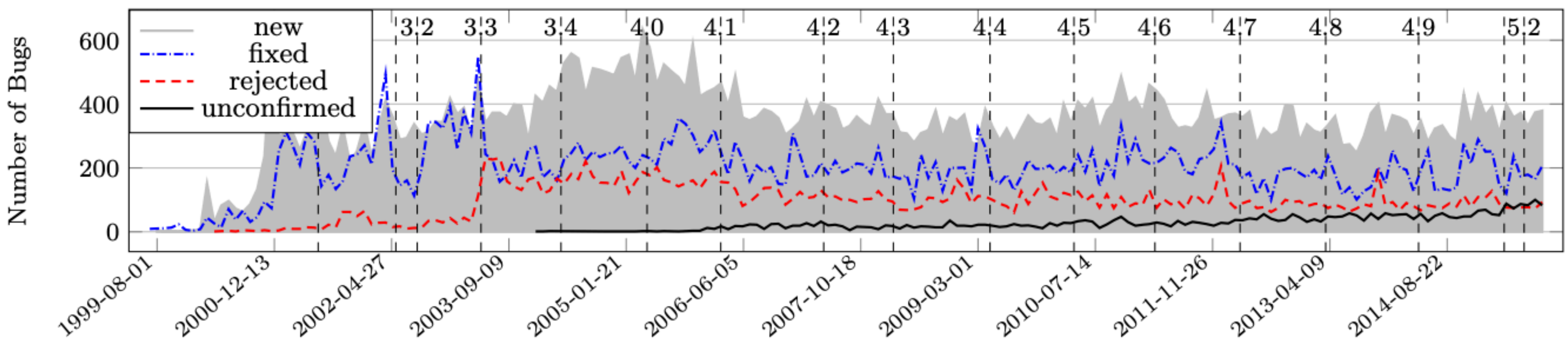
- Paper + implementation by Hawblitzel et al.
- Built four verified apps using Dafny (+ other tools)
  - Notary app securely assigns timestamps to documents
  - Password hasher
  - Differentially-private database
  - Multi-user trusted counter
- Also implemented code used by apps
  - Libraries: Ints, arrays, strings, …
  - Cryptographic tools
  - Network driver
  - …

Ironclad Apps

- Verified a host of correctness, security properties
  - Remote equivalence: "that to a remote party the app's implementation is *indistinguishable* from the app's high-level abstract state machine."
  - No code injection
  - No buffer overflow
  - Correct crypto implementations
  - No data leaks
  - …
- Nontrivial effort
  - ~6K lines of implementation
  - ~30K lines of proof annotations
  - ~3 person-years

CompCert

- Unfortunately, compilers are full of bugs



Number of Bug Reports for GCC compiler, Sun et al. ISSTA '16

- The CompCert project produced a formally verified C compiler
- Proves: the executable code produced by the compiler behaves exactly as specified by the semantics of the source C program

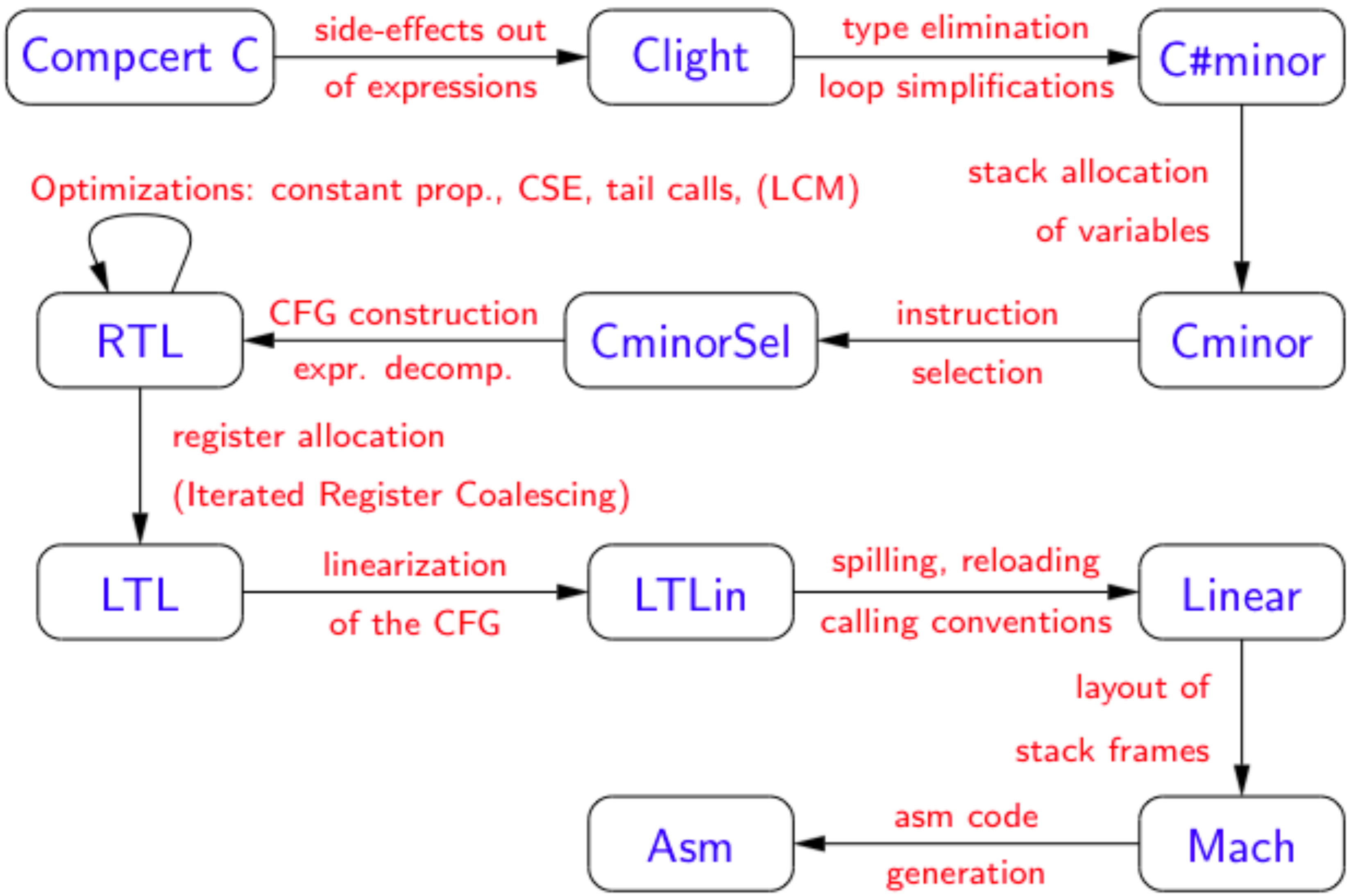
CompCert

- Compiler entirely written and verified using Coq, a programming language and proof assistant
  - Coq helps write, and mechanically checks, formal proofs

- High-level idea: Formally define source and target languages, prove compiler preserves semantics
  - For programs source S, target T, compiler C, prove:
  - $\forall$ input i: T = C(S) $\Rightarrow$ S(i) = T(i)

- Do this using many intermediate languages
  - Makes proofs easier

CompCert

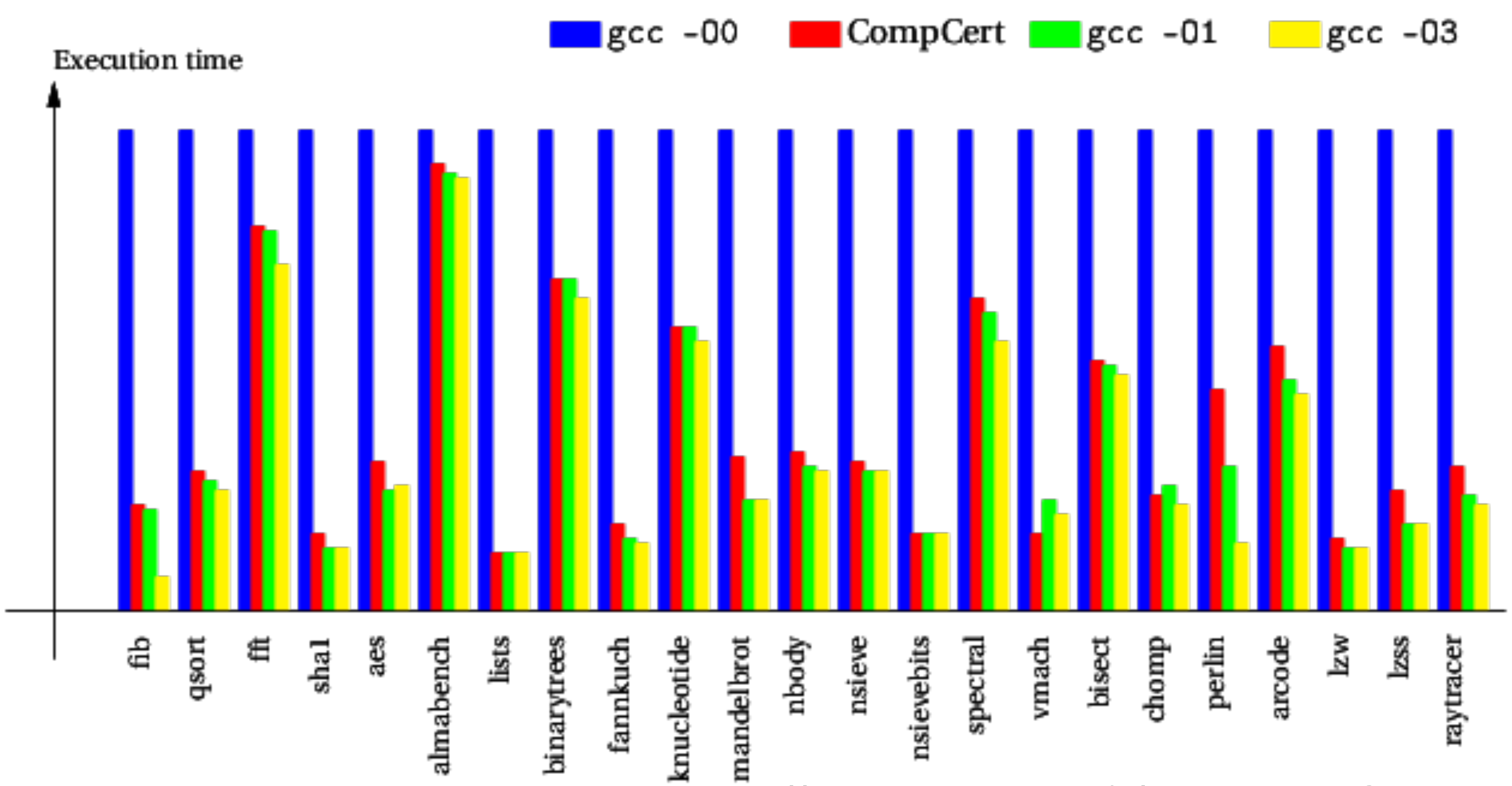


source: http://compcert.inria.fr/compcert-C.html

CompCert

- A major undertaking
  - ~6 person-years of effort
  - ~100,000 lines of Coq code

- Uses many optimizations, making its runtime competitive in practice:



source: http://compcert.inria.fr/compcert-C.html

More Successes in Verification

- seL4: A fully verified operating system kernel
  - Written in C, proven correct using proof assistant Isabelle
- *Refinement type* systems
  - Extend types with expressive predicates
  - e.g., `Integer x { x > 0 }`
  - Have been applied to Haskell, Racket, JavaScript, Ruby, and more
- Formally verified radiotherapy machine
  - In use at University of Washington Medical Center!
- …and many more!

Links

- Dafny tutorial
- Z3 tutorial
- Software Foundations: a textbook, written in Coq, teaching the basics of proofs about programs