COMP163 HW Assignment 1: Due Thursday, September 21, 2023, 11:59PM

Reading: Chapter 1 in the Text. Three sets of 2D convex hull notes and pp. 66-72 in "Computational Geometry: A User's Guide", all available by links located within the item "Lecture Notes" at the bottom of the website:

`https://www.cs.tufts.edu/comp/163`

Please consult other references as interest dictates.

General Information: Please type up your homework solutions (directions on how to use latex are included on the webpage) and submit the pdf – it is OK to hand-draw figures and "drop" them into the typed document. When describing an algorithm, do not forget to analyse its running time and explain why the algorithm is correct. Although we hope that you will discuss these problems in the preliminary stages with others, work submitted should be done individually and written in your own words **as clearly and succinctly as possible**. If you have any discussions with others (students, friends, TAs, faculty, ...) relative to a homework problem or if you gain information from a written (or video/audio) source other than your own notes from lecture, you are expected to identify your collaborator/source.

Problems:

1. Left Turn and Convexity:
   (a) Given points $A = (x_a, y_a)$, $B = (x_b, y_b)$, and $C = (x_c, y_c)$, the determinant $D =$

   $$\begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix}$$

   has the value $x_a y_b + x_c y_a + x_b y_c - x_c y_b - x_b y_a - x_a y_c$. This value equals twice the signed area of triangle ΔABC where the sign is + if and only if A, B, C appear in counterclockwise order on the boundary of ΔABC. In other words, A, B, C forms a left turn if and only if $D > 0$. Use high school mathematics to derive and verify this fact.

   (b) Given a polygon P specified by a circularly linked list of its n vertices in order, provide an algorithm to test whether the polygon P is convex and prove its correctness.

2. Un-ordered Divide and Conquer Convex Hull:
   (a) Let P_1 and P_2 be n_1- and n_2-vertex convex polygons, respectively, with no vertices in common. Prove that the number of lines of support common to P_1 and P_2 is at most $2 * min\{n_1, n_2\}$ and that this bound is achievable. [How many common lines of support can they have if P_1 and P_2 do not intersect? How many if P_1 is interior to P_2]

   (b) Given two arbitrary convex polygons P_1 and P_2 with n_1 and n_2 vertices each (their boundaries may intersect one, two, or more times; they may be disjoint; one may be contained within the other), specify as efficient an algorithm as you can to compute the convex hull of $P \cup Q$. Analyse its complexity.

   (c) Use this algorithm to generate a divide-and-conquer algorithm without presorting for finding the convex hull of an arbitrary set of n points. Analyse complexity.

3. TO BE STARTED AS AN IN-CLASS PROBLEM-SOLVING EXERCISE AND THEN WRITTEN UP INDIVIDUALLY
   Given a set S of n planar points in general position and in arbitrary order, provide and analyse the most efficient algorithm that you can for constructing:

   - a simple monotone polygon R whose vertices are exactly the points in S and which contains the line segment between the point in S of smallest x-coordinate and the point in S of largest x-coordinate. [Can you do this in $\Theta(n \log n)$ time?]
   - a simple starshaped polygon P whose vertices are exactly the points in S for which the point of S of smallest x-coordinate "sees" every point of P. [Can you do this in $\Theta(n \log n)$ time?]

   Now assume that you are given a series of query points q that are NOT in S. For each of P and R, provide and analyze an algorithm that will test whether each query point q is interior or exterior to that polygon. [Can you do this in $\Theta(\log n)$ time?]