COMP163 Homework Assignment 4
Due Thursday, November 2, 2023

Reading:
Read the chapter on Voronoi diagrams/Delaunay triangulations in your text, if you haven't already. Read the comparable sections in the lecture notes. Begin reading in the handouts and in the text about higher dimension convex hulls.

Problems:

1. The *Gabriel Graph*, GG, of a set S in E^2 is defined as follows: let disk (p_i, p_j) be the circle having $p_i\bar{p}_j$ as a diameter; the Gabriel Graph of S has an edge between p_i and p_j in S if and only if disk (p_i, p_j) contains no point of S in its interior. Show that p_i and p_j have an edge in GG if and only if this edge both appears in the Delaunay triangulation of S and crosses its dual Voronoi edge. Use this to show that the Gabriel graph for any set S can be constructed in $O(n \log n)$ time, and also show the optimality of your algorithm. Give an example of a set S for which the Gabriel Graph and the Delaunay triangulation are NOT the same.

2. Given a set S of n points in the Euclidean plane and a tolerance k:
   (a) describe and analyse an algorithm to report, for each query point q in the plane,the largest circle centered at q that contains no point of S in its interior.
   (b) describe and analyse an algorithm to report, for each pair of query points p, q, a path from p to q contained entirely within the convex hull of S such that no point on the path is closer than k to any point of S, if such a path exists.

3. Submit a rough plan of your programming/visualization project.

4. OPTIONAL: In the Euclidean space E^d of coordinates $x_1, x_2, ...x_d$, for any real number p such that $1 \leq p \leq \infty$, the L_p-distance of two points q_1 and q_2 is given by the norm $d_p(q_1, q_2) = (\sum_{j=1}^{d}(|x_j(q_1) - x_j(q_2)|)^p)^{1/p}$. So far, we have been studying the L_2-distance in E^2 (the plane).

   Draw a set of 4-5 points at integer grid points and work out each of the parts below on this small sample. Then generalize if possible.
   (a) In the plane, characterize the Voronoi diagram of a set of N points in the L_1-metric.
   (b) Solve the same problem for the L_∞-metric.
   (c) What is the relationship between the Voronoi diagram in the L_1-metric and that in the L_∞-metric?