COMP 181

Lecture 3

Lexical analysis

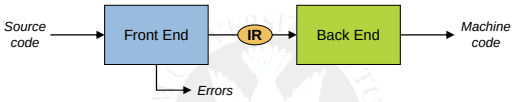
Tuesday, Sept. 12, 2006

Prelude

- What happened on August 14, 2003?
  - 2003 North American Blackout
  - 50 million people without power

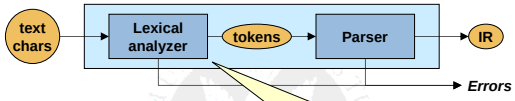
- How did it happen?
  - Cascading power line failure
    - More power on a line causes it to heat up
    - Heating causes the metal conductor to expand
    - The power line sags, hits a tree, fails
- Why didn't power co.'s respond more quickly?
  - No alarm sounded on early failures
  - Software bug!

Big picture



- Front end responsibilities
  - Check that the input program is legal
    - Check syntax and semantics
    - Emit meaningful error messages
  - Build IR of the code for the rest of the compiler

Front end design



Terms "Scanner" and "lexical analyzer" used interchangeably

- Two part design
  - Scanner
    - Reads in characters
    - Classifies sequences into words or *tokens*
  - Parser
    - Checks sequence of tokens against grammar
    - Creates a representation of the program (AST)

Scanner and parser

- Why separate scanner and parser?
  - Simplifies the implementation
  - Parsing is fundamentally harder
    - Word classification is easier – make it fast
    - Speed up parsing by working with tokens

Lexical analysis

- The input is just a sequence of characters.
  Example:

```
if (i == j)
  z = 0;
else
  z = 1;
```

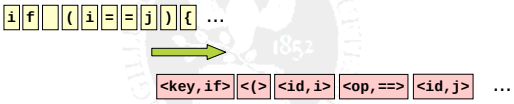
- More accurately, the input is:

```
\tif (i == j)\n\t\tz = 0;\n\telse\n\t\tz = 1;
```

- **Goal**: Partition input string into substrings
  - And classify them according to their role

Scanner

- Responsibilities
  - Read in characters
  - Produce a stream of tokens



  - Token has a type and a value

Hand-coded scanner

- Explicit test for each token
  - Read in a character at a time
  - Example: recognizing keyword "if"

```
c = readchar();
if (c != 'i')
  error();
else {
  c = readchar();
  if (c != 'f')
    error();
  else
    return IF_TOKEN;
}
```


Hand-coded scanner

- What about other tokens?

  Example: "if" is a keyword, "if0" is an identifier

```
c = readchar();
if (c != 'i') { other tokens… }
else {
  c = readchar();
  if (c != 'f') { other tokens… }
  else {
    c = readchar();
    if (c not alpha-numeric) {
      putback(c);
      return IF_TOKEN; }
    while (c alpha-numeric) { build identifier }
```


Hand-coded scanner

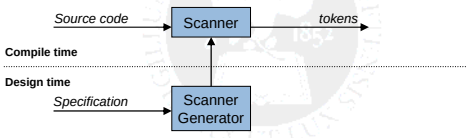
Problems:

- Many different kinds of tokens
  - Fixed strings (keywords)
  - Special character sequences (operators)
  - Tokens defined by rules (identifiers, numbers)
- Tokens overlap
  - "if" and "if0" example
  - "=" and "=="
- Coding this by hand is too painful!

  Getting it right is a serious concern

Scanner construction

- Goal: automate process
  - Avoid writing scanners by hand
  - Leverage the underlying theory of languages



Outline

Problems we need to solve:

- Scanner specification language
  - How to describe parts of the input language
- The scanning mechanism
  - How to break input string into tokens
- Scanner generator
  - How to translate from (1) to (2)
- Ambiguities
  - The need for ***lookahead***

Problem 1: Describing the scanner

- We want a high-level language **D** that
  1. Describes lexical components, and
  2. Maps them to tokens (determines type)
  3. ***But*** doesn't describe the scanner algorithm itself !
- Part 3 is important
  - Allows focusing on what, not on how
  - Therefore, D is sometimes called a specification language, not a programming language
- Part 2 is easy, so let's focus on Parts 1 and 3

Token examples

- Keyword
  - Exact sequence of characters
- Identifier
  - Sequence of letters or numbers, starting with a letter
- Number
  - Sequence of digits
- Whitespace
  - Sequence of space, tab, carriage-return

Specifying tokens

- Many ways to specify them
- **Regular expressions** are the most popular
  - REs are a way to specify *sets of strings*
  - Examples:
    - 'a' – denotes the set {"a"}
    - 'a'|'b' – denotes the set {"a", "b"}
    - 'a''b' – denotes the set {"ab"}
    - 'a''b'* – denotes the set {"a", "ab", "abb", "abbb", … }
- Why regular expressions?
  - Easy to understand
  - Strong underlying theory
  - Very efficient implementation

May specify sets of infinite size

Formal languages

- ***Def***: a language is a set of strings
  - Alphabet Σ : the character set
  - Language is a set of strings over alphabet
- Each regular expression denotes a language
  - If A is a regular expression, then L(A) is the set of strings denoted by A
  - Examples: given Σ = {'a', 'b'}
    - A = 'a' L(A) = {"a"}
    - A = 'a'|'b' L(A) = {"a", "b"}
    - A = 'a''b' L(A) = {"ab"}
    - A = 'a''b'* L(A) = {"a", "ab", "abb", "abbb", … }

Building REs

- Regular expressions over Σ
- Atomic REs
  - ε is an RE denoting empty set
  - if a is in Σ, then a is an RE for {a}
- Compound REs
  - if *x* and *y* are REs then:
    - *xy* is an RE for *L(x)L(y)* *Concatentation*
    - *x|y* is an RE for *L(x)* ∪ *L(y)* *Alternation*
    - *x** is an RE for *L(x)** *Kleene closure*

Using regular expressions

- Concatenation: build up words
- Kleene closure: repetition
- Alternation: collect sets of words
- Back to our language problem…

Keywords

- Exact strings: “if” or “for” or “while”
  - Singleton sets
  - Built up using concatenation

L(‘i’ ‘f’) = {“if”}
L(‘f’ ‘o’ ‘r’) = {“for”}

  - We can abbreviate ‘i’ ‘f’ as ‘if’

‘if’ | ‘for’ | ‘else’ | …

Integers

- Integer: non-empty sequence of digits

digit = ‘0’ | ‘1’ | ‘2’ | ‘3’ | ‘4’ | ‘5’ | ‘6’ | ‘7’ | ‘8’ | ‘9’
integer = digit digit*

- Another abbreviation:

A+ ⇔ AA*

Identifiers

- Identifier: string of letters or numbers starting with a letter

letter = ‘a’ | ‘b’ | … | ‘z’ | ‘A’ | … | ‘Z’
identifier = letter (letter | digit) *

- Is this the same as **(letter|digit)+** ?
- How about **(letter*|digit*)** ?

Other examples

- Numbers

int = (‘+’|’-’| ε) digit+
decimal = int ‘.’ digit+
real = (int | decimal) (‘E’(‘+’|’-’| ε)digit+)

- What about IP addresses?

ip = digit+ ‘.’ digit+ ‘.’ digit+ ‘.’ digit+

  - Is this right?
  - Can we be more precise?

Outline

Problems we need to solve:

- Scanner specification language ***DONE***
  - How to describe parts of the input language
- The scanning mechanism ⇐
  - How to break input string into tokens
- Scanner generator
  - How to translate from (1) to (2)
- Ambiguities
  - The need for ***lookahead***

Overview of scanning

- How do we recognize strings in the language?

Every RE has an equivalent finite state automaton that recognizes its language
(Actually, more than one)

  - Idea: scanner simulates the automaton
    - Read characters
    - Transition automaton
    - Return a token if automaton accepts the string

Finite Automata

- Regular expressions = specification
- Finite automata = implementation
- A finite automaton consists of
  - An input alphabet Σ
  - A set of states S
  - A start state n
  - A set of accepting states $F \subseteq S$
  - A set of transitions state $\rightarrow^{input}$ state

Finite Automata

- Transition

  $s_1 \rightarrow^a s_2$

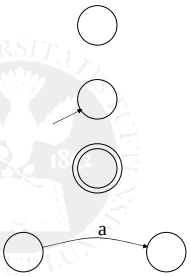
- Is read

  In state s_1 on input "a" go to state s_2

- If end of input
  - If in accepting state => accept
  - Otherwise => reject

Finite Automata State Graphs

- A state
- The start state
- An accepting state
- A transition



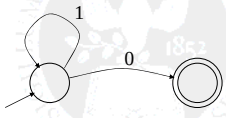
A Simple Example

- A finite automaton that accepts only "1"

- A finite automaton accepts a string if we can follow transitions labeled with the characters in the string from the start to some accepting state

Another Simple Example

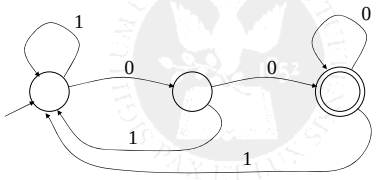
- FA accepts any number of 1's followed by a single 0
- Alphabet: {0,1}



- Check that "1110" is accepted but "110…" is not

And Another Example

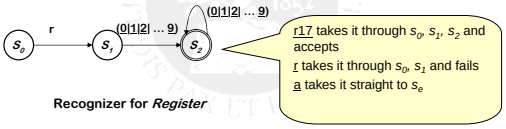
- Alphabet {0,1}
- What language does this recognize?



"Realistic" example

- Consider the problem of recognizing machine register names

  Register → r (0|1|2| … | 9) (0|1|2| … | 9)*



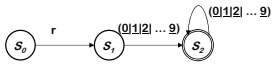
Recognizer for *Register*

Implementation

- Finite automaton
  - States, characters
  - State transition δ uniquely determines next state

  ⇨ Automaton is ***deterministic***
- Next character function
  - Reads next character into buffer
  - (May compute ***character class*** by fast table lookup)
- Transitions from state to state
  - Implement δ as a table
  - Access table using current state and character

Example



Turning the recognizer into code

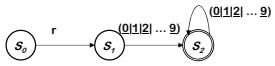
δ	r	0,1,2,3,4, 5,6,7,8,9	*All others*
s_0	s_1	s_e	s_e
s_1	s_e	s_2	s_e
s_2	s_e	s_2	s_e
s_e	s_e	s_e	s_e

Table encoding RE

```
Char ← next character
State ← s0
while (Char ≠ EOF)
    State ← δ(State,Char)
    Char ← next character
if (State is a final state )
    then report success
    else report failure
```

Skeleton recognizer

Example



Adding actions

δ	r	0,1,2,3,4, 5,6,7,8,9	*All others*
s_0	s_1 *start*	s_e error	s_e error
s_1	s_e error	s_2 add	s_e error
s_2	s_e error	s_2 add	s_e error
s_e	s_e error	s_e error	s_e error

Table encoding RE

```
Char ← next character
State ← s0
while (Char ≠ EOF)
    State ← δ(State,Char)
    perform specified action
    Char ← next character
if (State is a final state )
    then report success
    else report failure
```

Skeleton recognizer

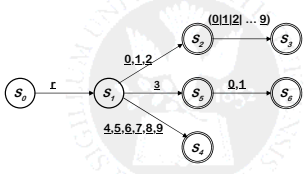
What if we need a tighter specification?

- r *Digit Digit** allows arbitrary numbers
  - Accepts r00000
  - Accepts r99999
  - What if we want to limit it to r0 through r31 ?
- Write a tighter regular expression
  - *Register* → r ((0|1|2) (*Digit* | ε) | (4|5|6|7|8|9) | (3|30|31))
  - *Register* → r0|r1|r2| … |r31|r00|r01|r02| … |r09
- Produces a more complex DFA
  - Has more states
  - Same cost per transition
  - Same basic implementation

Tighter register specification

- The DFA for

  Register → r ((0|1|2) (*Digit* | ε) | (4|5|6|7|8|9) | (3|30|31))



- Accepts a more constrained set of registers
- Same set of actions, more states

Tighter register specification

δ	r	0,1	2	3	4-9	All others
s_0	s_1	s_e	s_e	s_e	s_e	s_e
s_1	s_e	s_2	s_2	s_5	s_4	s_e
s_2	s_e	s_3	s_3	s_3	s_3	s_e
s_3	s_e	s_e	s_e	s_e	s_e	s_e
s_4	s_e	s_e	s_e	s_e	s_e	s_e
s_5	s_e	s_6	s_e	s_e	s_e	s_e
s_6	s_e	s_e	s_e	s_e	s_e	s_e
s_e	s_e	s_e	s_e	s_e	s_e	s_e

Runs in the same skeleton recognizer

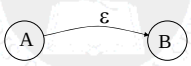
Table encoding RE for the tighter register specification

REs and DFAs

- Key idea:
  - Every regular expression has an equivalent DFA that accepts only strings in the language
- Problem:
  - How do we construct the DFA for an arbitrary regular expression?
  - Not always easy

Example

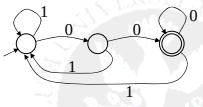
- What is the RE for a(a|ε)b?
- Need ε moves



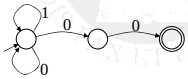
- Transition A to B without consuming input!

Another example

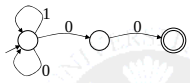
- Remember this DFA?



- We can simplify it as follows:



A different kind of automaton



- Accepts the same language
  Actually, it's easier to understand!
- What's different about it?
  - Two different transitions on '0'
  - This is a ***non-deterministic finite automaton***

DFAs and NFAs

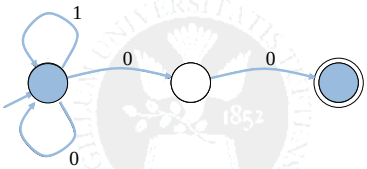
- Deterministic Finite Automata (DFA)
  - One transition per input per state
  - No ε-moves
- Nondeterministic Finite Automata (NFA)
  - Can have multiple transitions for one input in a given state
  - Can have ε-moves

Execution of Finite Automata

- A DFA can take only one path through the state graph
  - Completely determined by input
- NFAs can choose
  - Whether to make ε-moves
  - Which of multiple transitions for a single input to take

Acceptance of NFAs

- An NFA can get into multiple states



- Input: 1 0 0
- Rule: NFA accepts if it can get in a final state

Non-deterministic finite automata

- An NFA accepts a string *x* iff ∃ a path though the transition graph from s_0 to a final state such that the edge labels spell *x*

  (Transitions on ε consume no input)
- To "run" the NFA, start in s_0 and ***guess*** the right transition at each step
  - Always guess correctly
  - If some sequence of correct guesses accepts x then accept

Why do we care about NFAs?

- Simpler, smaller than DFAs
- More importantly:
  - Need them to support all RE capabilities
  - Systematic conversion from REs to NFAs
  - Need ε transitions to connect RE parts
- Problem: how to implement NFAs?
  - How do we guess the right transition?
  - Multiple states: what about memory usage?

Relationship between NFAs and DFAs

- DFA is a special case of an NFA
  - DFA has no ε transitions
  - DFA's transition function is single-valued
  - Same rules will work
- DFA can be simulated with an NFA *(obvious)*
- NFA can be simulated with a DFA *(less obvious)*
  - Simulate sets of possible states
  - Possible exponential blowup in the state space
  - Still, one state per character in the input stream

Automatic scanner construction

- To convert a specification into code:
  1. Write down the RE for the input language
  2. Build a big NFA
  3. Build the DFA that simulates the NFA
  4. Systematically shrink the DFA
  5. Turn it into code
- Scanner generators
  - Lex and Flex work along these lines
  - Algorithms are well-known and well-understood
  - Key issue is interface to parser *(define all parts of speech)*
  - You could build one in a weekend!

Automatic scanner construction

RE → NFA *(Thompson's construction)*
- Build an NFA for each term
- Combine them with ε-moves

NFA → DFA *(subset construction)*
- Build the simulation

DFA → Minimal DFA
- Hopcroft's algorithm

DFA → RE *(Not part of the scanner construction)*
- All pairs, all paths problem
- Take the union of all paths from s_0 to an accepting state

Next time...

- RE *-to-* NFA *-to-* DFA *-to-* scanner
- Algorithms (yikes!)
- Programming assignment 1